Quantum Aether

Volume VII:Operational Einstein:

A Working Unified Spectral Field Theory.

Numerics, Gates, and DoV Audits

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October 26, 2025

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Part I

Operational Unification: From MSP to a Working Unified Field

Chapter 1

Charter and Freeze of Sovereign Inputs

Prelude. Volume VII upgrades the *Quantum–Aether* program from a conceptual, gate-driven framework into a *working* Unified Spectral Field Theory (USFT). On a declared DoV, we compress Vols. I–VI into a single operational stack that (i) fixes the minimal MSP inputs, (ii) enforces the four gates S/G/TQ/D as automated acceptance criteria, and (iii) ships public artifacts (HDF5 seeds, notebooks, CI logs) enabling independent reproduction of every number and certificate.

This opening chapter states the charter and freezes the *sovereign inputs* that the rest of the volume must treat as read-only: coupling scales and topological indices entering MSP, the operational DoV (scales, backgrounds, boundary triggers), and the gate legend with their quantitative exit metrics. Success here is *not* a narrative claim but a checklist: baseline gate certificates pass on reference backgrounds; the DoV manifest is published; and the seeds/environments needed to reproduce the baseline are pinned and hashed.

Downstream chapters build on this freeze: the spectral action and geometry, well-posed PDEs and numerical solvers, BRST quantization and local GKSL embeddings, the spectral-flooding $\Rightarrow$ Born limit, recovery of GR/SM, and a portfolio of falsifiable predictions with decision windows. The charter therefore functions as the contract between principle and execution: it binds predictions to inputs, gates to proofs, and numerics to public fingerprints.

1.1 Objectives, Scope, and Success Metrics

This chapter binds the *execution layer* of Volume VII to three compact commitments: a working Unified Spectral Field Theory (USFT) defined on a declared DoV, an operational scope that compresses Vols. I–VI into a single stack with enforced gates, and measurable success criteria with public artifacts.

- **Objective.** Deliver a *working* Unified Spectral Field Theory (USFT) that is self-consistent on a declared DoVand produces falsifiable, numerically tight predictions with public artifacts.
- **Scope.** Compress Vols. I–VI (MSP, geometry, quantization, verification, audits) into a single operational stack; enforce Gates S/G/TQ/D.
- **Success metrics.** (i) At least one high-precision prediction with a *kill window*; (ii) full DoVaudit; (iii) independent reproduction from HDF5/CI bundle.

Operational commitments (summary).

(C1) **Freeze.** Publish the MSP* inputs and DoV manifest (scales, backgrounds, boundary triggers); pin environments and seeds.

(C2) **Gates.** Automate S/G/TQ/D with quantitative exit checks attached to each result.

(C3) **Artifacts.** Ship an HDF5 bundle (v7), notebooks/API, and CI logs enabling *one-command* rebuild and verification.

Acceptance 1.1. Success & Exit Criteria (chapter-level) A1: Formal statement of Objective/Scope with precise DoV and gate definitions.
A2: Public `msp_v7.h5` seed + environment lockfile + CI recipe for replay.
A3: Baseline gate certificates pass on reference backgrounds (Minkowski/FRW), and at least one prediction ships with a declared kill window and uncertainty budget.

Quantitative KPIs.

The following key performance indicators (KPIs) are the minimal numeric targets for Volume VII. Each KPI is accompanied by an acceptance test and a gate link.

KPI	Quantity	Acceptance test	Gate
P1	$\sin^2\theta_W(M_Z)$	Prediction within stated 1σ band; ellipsoid $\chi^2 \le \chi^2_{p,2}$	D
P2	(g_1, g_2, g_3) at $\mu_{\rm ref}$	Inside reference-scale acceptance ellipsoid; RG continuity certified	S/D
P3	(S, T, U)	Global-fit residuals inside window $\mathcal{W}_{STU}$	D
P4	EDM floors $d_f^{\rm floor}$	One-sided: $\lvert d_f\rvert^{95\%}_{\rm exp} \ge d_f^{\rm floor}(1-\delta_{\rm had})$	D
P5	DI/No-signalling	CHSH operator norm $\le 2\sqrt{2} + \delta_{\rm slack}$; DI vector feasible	TQ/D
S1	Cone LMI	$\mu_{\min} \ge 0$ across DoV tiles; margin report included	G
S2	Hyperbolicity	Symmetrizable form; $v_{\rm front} \le$ cEff; CFL-convergent	S
Q1	CPTP margins	Choi $\lambda_{\min} \ge -\epsilon_{\rm num}$; TP within $\delta_{\rm TP}$	TQ

Definition (Working USFT on DoV).

A *working* USFT in the sense of Volume VII is a theory stack for which: (i) cone/forward positivity and locality hold on the declared DoV, (ii) the quantum channel layer is CPTP

with certified margins and admits local GKSL (or CP time-local embeddings under colored noise), (iii) at least one prediction with an explicit kill window is produced and shipped as a public, replayable artifact, and (iv) all results pass the gate suite S/G/TQ/D under reproducible CI.

Checklist 1.1: Chapter deliverables

- **Manifest.** MSP* inputs and DoV (scales, backgrounds, boundary triggers) published with hashes.
- **Gate legend.** Quantitative definitions and thresholds for S/G/TQ/D.
- **Replay.** `make replay` regenerates baseline certificates and one prediction with its window.

1.2 Freeze: MSP and Minimal Inputs

Purpose. This section *freezes* the minimal sovereign inputs of the Master Spectral Principle (MSP) and publishes the operational DoV together with gate-level exit metrics. All downstream results in this volume must treat these inputs as read-only and reference the manifest by hash.

MSP* set.

We fix the minimal parameter vector

$$\boldsymbol{\theta}_{\mathrm{MSP}^*} := \Big(\kappa_G,\ \mathcal{N}_u,\ \mathcal{N}_d,\ \mathcal{N}_\ell;\ \text{topological indices}\Big),$$

where κ_G sets the geometric normalization, $\mathcal{N}_{u,d,\ell}$ are counting factors for matter families, and the topological indices include the relevant fiber invariants (e.g. Chern numbers, Stiefel–Whitney data) used by the spectral action. The freeze requires a public *manifest* describing the inputs, their units, and the exact keys used in the artifacts.

Symbol	Meaning	Role in stack	Units
κ_G	Geometric gauge normalization	Sets cone/forward coefficients and Lie-metric scale in coupling relations	dimensionless
$\mathcal{N}_u$	Up-type family count/weight	Enter overlap/Yukawa textures and thresholds	dimensionless
$\mathcal{N}_d$	Down-type family count/weight	As above	dimensionless
$\mathcal{N}_\ell$	Lepton family count/weight	As above	dimensionless
Top. indices	Fiber/bundle invariants	Fix charge quantization and anomaly inflow bookkeeping	–

DoV declaration.

The Domain of Validity is a tuple

$$\mathrm{DoV} \equiv \Big(\mathcal{B},\ \mathcal{S};\ \mathcal{T}\Big),$$

with $\mathcal{B}$ the set of backgrounds (e.g. Minkowski, FRW slices), $\mathcal{S}$ the scale ranges (energy, length, time) and detector regimes, and $\mathcal{T}$ the *boundary triggers*. Triggers implement automated end of validity when any of the following occurs on a declared background:

$$\Delta_{\mathcal{L}} \downarrow 0 \quad \text{(gap closure)}, \qquad \lambda_{\min}\Big(J(\Lambda_t)\Big) < 0 \quad \text{(CP loss)}, \qquad \mu_{\min}\Big(\mathsf{C}(n)\Big) < 0 \quad \text{(cone violation)}. \tag{1.1}$$

Here $\Delta_{\mathcal{L}} > 0$ is the Liouvillian mixing gap on pointer sectors, $J(\Lambda_t)$ the Choi matrix of the effective channel (or of a CP time-local embedding under colored noise), and $\mathsf{C}(n)$ the gravity-cone LMI block evaluated on unit covectors n of (M, g_{eff}). The declared locality radius $L_{\mathrm{loc}} > 0$ is part of the manifest and bounds admissible system–meter couplings.

Gates.

S (stability/positivity): principal symbol is symmetrizable hyperbolic and the quadratic energy form is positive; forward-limit EFT blocks are positive semidefinite. Formally, for some positive H, $HA^0 \succ 0$ and HA^i are symmetric, and the dispersion polynomial has real characteristics (no gradient ghosts).

G (causal geometry): cone LMIs $\mu_{\min}(\mathsf{C}(n)) \geq 0$ for all unit n, with locality radius L_{loc} declared and respected by channels/instruments.

TQ (quantum/CP): channel layer is CPTP with margins:

$$J(\Lambda) \succeq 0, \qquad \mathrm{Tr}_B\, J(\Lambda) = \mathbb{I}_A, \qquad \lambda_{\min}(J) \geq -\epsilon_{\mathrm{num}}, \quad \|\,\mathrm{Tr}_B\, J - \mathbb{I}_A\| \leq \delta_{\mathrm{TP}}.$$

Under colored noise, a CP time-local embedding exists on the DoV.

D (decision windows): each prediction ships with a predeclared acceptance region $\mathcal{W}$ and a kill rule, e.g. an ellipsoid $(\widehat{O} - O_0)^\top \Sigma^{-1} (\widehat{O} - O_0) \leq \chi^2_{p,\alpha}$ or a one-sided bound (EDM floors, DI feasibility vectors).

Manifest (public).

The freeze publishes a machine-readable manifest with keys minimally including: `/msp/inputs` (MSP* tuple + units), `/dov` (backgrounds, scales, L_{loc}, triggers), `/gates` (thresholds and tolerances), and `/provenance` (hashes, environment lockfiles).

Acceptance 1.2. A1–A3 **A1** Formal statement of MSP* and DoV with quantitative gate definitions and boundary triggers.
A2 Public manifest and seeds (HDF5 bundle + environment lockfile) with hashes enabling replay.
A3 Baseline gate certificates pass on reference backgrounds (Minkowski/FRW): S hyperbolicity & forward positivity, G cone LMIs with declared L_{loc}, TQ CPTP margins (or CP embeddings under color), and D decision windows attached to at least one prediction.

Part II

Unified Spectral Action and Geometry

Chapter 2

Spectral Lagrangian and Variational Structure

Prelude. This chapter fixes the *working* form of the spectral action that drives the Unified Spectral Field Theory (USFT) on its declared DoV. The action is a local functional on (M, g_{eff}) built from the invariant, unit-bearing descriptors

$$\mathcal{S} = \{\Phi, \rho, \eta, \chi^{\mu}, T\},$$

together with a spectral derivative $D_{\mu}(\mathcal{S})$ and a connection $\omega_{\mu}(\mathcal{S})$ that transport fiber data and couple them to the emergent geometry. Variation of the action yields Euler–Lagrange equations and Noether currents; these will be organized so that hyperbolicity/positivity (S) and cone-causality (G) are checkable from the principal symbol, while quantum and dissipation requirements (TQ/D) can be attached to the channel layer introduced in later chapters.

The chapter unfolds as follows. We first state the action density and its unit conventions, then list the fields/descriptors and their transformation rules. Next, we introduce the spectral derivative D_{μ}, the fiber connection ω_{μ}, and the induced effective metric $g_{\text{eff}}[\mathcal{S}]$ that shapes causal cones and locality radius L_{loc}. Finally, we record the variational structure (Euler–Lagrange equations, stress tensor, and Noether identities), laying down the anchors needed for the PDE well-posedness and gauge/geometry closure that follow.

Notation. Tr is the fiber trace; $\mathbb{I}$ the identity; Spec denotes the spectral content of operators. Bold symbols denote stacks of fields, and $\nabla^{(g_{\text{eff}})}$ denotes the Levi–Civita derivative of g_{eff}. Unless stated otherwise, all integrals and Hodge duals are taken with respect to g_{eff} and the declared DoV background.

2.1 Action density and units

We write the conservative (Hamiltonian) sector of the Unified Spectral Field Theory (USFT) as an action on (M, g_{eff})

$$S[\mathcal{S}, g_{\text{eff}}, \omega] \;=\; \int_M \mathrm{d}^4x \sqrt{-\det g_{\text{eff}}}\; \mathcal{L}(\mathcal{S}, \partial\mathcal{S}; g_{\text{eff}}, \omega), \qquad \mathcal{S} = \{\Phi, \rho, \eta, \chi^\mu, T\}, \tag{2.1}$$

with Lagrangian density split into transparent blocks

$$\mathcal{L} \;=\; \mathcal{L}_{\text{geom}}(g_{\text{eff}}) + \mathcal{L}_{\text{kin}}(\mathcal{S}, \partial\mathcal{S}; g_{\text{eff}}, \omega) + \mathcal{L}_{\text{pot}}(\mathcal{S}) + \mathcal{L}_{\text{YM}}(\omega; g_{\text{eff}}) + \mathcal{L}_{\text{constr}}(\mathcal{S}; g_{\text{eff}}). \tag{2.2}$$

Only the conservative sector appears in S; dissipation/measurement channels are introduced later by a Rayleigh/GKSL layer (Part III, IV). The blockwise form (2.2) is chosen so that the *principal symbol* and positivity checks for S/G are visible at a glance.

Unit system.

Fix reference scales (M_*, L_*, T_*) and let $[x^\mu] = L_*$, $[g_{\text{eff}}\mu\nu] = 1$, $[\mathcal{L}] = M_* L_*^{-1} T_*^{-2}$. We keep c_{eff} explicit when helpful; in numerics one may set $\hbar = c_{\text{eff}} = k_B = 1$ after unit audits. Table 2.1 records dimensions of the descriptors; coefficients below are written so that every term in $\mathcal{L}$ has the same units.

Symbol	Interpretation	Units
Φ	Spectral phase (order parameter)	dimensionless
ρ	Spectral density (mass/energy scale)	$M_* L_*^{-3}$ (or model-specific)
η	Spectral modulus (rigidity/viscous scale)	$M_* L_*^{-1} T_*^{-1}$
χ^μ	Director (timelike unit field)	L_*^0, with $g_{\text{eff}}\mu\nu\chi^\mu\chi^\nu = -1$
T	Temperature-like descriptor	$M_* L_*^2 T_*^{-2}$ (energy)
ω_μ	Fiber connection ($\mathfrak{iso}(\mathcal{F}, h)$-valued)	L_*^{-1}

Table 2.1: Descriptors and bookkeeping units used to build $\mathcal{L}$.

2.1.1 Fields and descriptors $(\Phi, \rho, \eta, \chi^\mu, T)$

The kinematic energy is quadratic in first derivatives and organized through a *kinetic matrix* $K_{AB}(\mathcal{S})$ acting on the field multiplet $S^A \in \{\Phi, \rho, \eta, T, \chi^\mu\}$:

$$\mathcal{L}_{\text{kin}} \;=\; \tfrac{1}{2} K_{AB}(\mathcal{S})\, g_{\text{eff}}^{\mu\nu}\, (D_\mu S^A)\, (D_\nu S^B) \;+\; \tfrac{1}{2} K_\chi(\mathcal{S})\, g_{\text{eff}}^{\mu\alpha} g_{\text{eff}}^{\nu\beta}\, \nabla_{[\mu}\chi_{\nu]}\, \nabla_{[\alpha}\chi_{\beta]}. \tag{2.3}$$

Here D_μ is the spectral derivative (defined below) and ∇ is the Levi–Civita covariant derivative of g_{eff}. The antisymmetric form for χ^μ penalizes vorticity/shear while leaving reparametrizations along χ^μ harmless; one may add a small longitudinal term $+\frac{\kappa_\parallel}{2}(\nabla_\mu\chi^\mu)^2$

if required by stability maps. The *constraint block* enforces normalization and gauge-fix choices:

$$\mathcal{L}_{\text{constr}} \;=\; \lambda_\chi \left(g_{\text{eff}}\mu\nu\chi^\mu\chi^\nu + 1 \right) \;+\; \text{(gauge-fixing terms as needed)}, \tag{2.4}$$

with a Lagrange multiplier λ_χ.

The *potential* collects analytic, unit-consistent invariants:

$$\mathcal{L}_{\text{pot}} \;=\; -U(\mathcal{S}) \;=\; -\Big[V_\Phi(\Phi;\rho,T) + V_\rho(\rho) + V_\eta(\eta) + V_T(T) + V_{\text{mix}}(\Phi,\rho,\eta,T;\chi^\mu\chi^\nu g_{\text{eff}}\mu\nu) \Big]. \tag{2.5}$$

A common choice is to keep V_{mix} to quadratic order in small departures from a reference background $(\bar\Phi, \bar\rho, \bar\eta, \bar T, \bar\chi^\mu)$, so that the Hessian of U contributes to the mass matrix in the principal symbol analysis of S.

Gate S anchor (principal symbol).

Linearizing S around a background solution yields for a Fourier mode with covector n_μ the principal symbol

$$\mathsf{P}(n)_{AB} \;=\; K_{AB}(\bar{\mathcal{S}})\, g_{\text{eff}}^{\mu\nu} n_\mu n_\nu,$$

whose positive semidefiniteness and real characteristics define the hyperbolicity window; these are the quantities audited in the stability maps of Part III.

2.1.2 Spectral derivative D_μ, connection ω_μ, metric g_{eff}

Spectral derivative.

Let $(\mathcal{F}, h)$ denote the internal fiber with metric h and isometry group $\mathrm{Iso}(\mathcal{F}, h)$. Fields carrying fiber indices transform in some representation R of the Lie algebra $\mathfrak{iso}(\mathcal{F}, h)$. The *spectral derivative* acts as a metric/gauge covariant derivative

$$D_\mu S^A \;=\; \nabla_\mu^{(g_{\text{eff}})} S^A \;+\; (\omega_\mu \cdot S)^A, \qquad \omega_\mu \in \mathfrak{iso}(\mathcal{F}, h), \tag{2.6}$$

with $(\omega_\mu \cdot S)^A = (\omega_\mu)^A{}_B S^B$ for nontrivial representations and $(\omega_\mu \cdot S)^A = 0$ for fiber scalars. For the phase field Φ one may allow a minimal coupling to an Abelian component of ω_μ, $D_\mu \Phi = \partial_\mu \Phi + q_\Phi\, \omega_\mu^{\mathrm{U}(1)}$.

Fiber connection and curvature.

The Ehresmann connection ω_μ induces curvature

$$\mathcal{F}_{\mu\nu} \;=\; \partial_\mu \omega_\nu - \partial_\nu \omega_\mu + [\omega_\mu, \omega_\nu] \in \mathfrak{iso}(\mathcal{F}, h), \tag{2.7}$$

and a Yang–Mills block

$$\mathcal{L}_{\mathrm{YM}} \;=\; -\frac{1}{4}\, g_{AB}(\mathcal{S})\, g_{\mathrm{eff}}^{\mu\alpha} g_{\mathrm{eff}}^{\nu\beta}\, \mathcal{F}^{A}_{\mu\nu}\, \mathcal{F}^{B}_{\alpha\beta}, \tag{2.8}$$

where $g_{AB}(\mathcal{S})$ is a positive fiber metric that may depend slowly on the descriptors (dimensionless after factoring the scale κ_G). The combination (2.8) is the anchor for the geometric-coupling relations developed later (Part 3).

Emergent metric g_{eff}.

The effective metric is a functional of the descriptors constrained by unit and symmetry requirements. A convenient ansatz around a reference background is

$$g_{\mathrm{eff}}\mu\nu \;=\; \alpha(\rho,\eta,T)\, g_{\mu\nu} + \beta(\rho,\eta,T)\, \chi_\mu\chi_\nu + \gamma(\rho,\eta,T)\, u_{(\mu}\chi_{\nu)} \;+\; \mathcal{O}(\delta\mathcal{S}^2), \tag{2.9}$$

where $g_{\mu\nu}$ is a fiducial background (used only for bookkeeping), u^μ a chosen frame if needed, and $\chi_\mu = g_{\mathrm{eff}}\mu\nu\chi^\nu$. Normalization $g_{\mathrm{eff}}\mu\nu\chi^\mu\chi^\nu = -1$ fixes one linear relation among (α,β,γ). In practice, coefficients are frozen by the MSP normalization (κ_G) and by cone audits (G), which require the *gravity cone* LMIs to be nonnegative for all unit covectors n_μ:

$$\mu_{\min}\Big(\mathsf{C}(n; g_{\mathrm{eff}}, \mathcal{S})\Big) \;\geq\; 0 \qquad \text{(for all unit } n_\mu). \tag{2.10}$$

Geometric block.

The purely geometric term is normalized by κ_G,

$$\mathcal{L}_{\mathrm{geom}} \;=\; \frac{1}{2\,\kappa_G}\, R(g_{\mathrm{eff}}) \;+\; \Lambda_{\mathrm{eff}}(\mathcal{S}), \tag{2.11}$$

with $R(g_{\mathrm{eff}})$ the Ricci scalar of g_{eff} and Λ_{eff} an effective vacuum piece (absorbing slow spectral variations). When one works on fixed backgrounds (Minkowski/FRW slices), (2.11) reduces to a constant and drops from the PDE principal symbol.

Summary of acceptance anchors.

The parameterization (2.3)–(2.11) is designed so that: (i) the kinetic matrix $K_{AB} \succeq 0$ and cone LMIs (10.9) realize S/G; (ii) the fiber block (2.8) aligns with the geometric determination of couplings developed later; and (iii) all coefficients carry explicit units relative to (M_*, L_*, T_*), simplifying nondimensionalization and CFL audits in Part 6.

2.2 Euler–Lagrange equations; Noether currents

We collect the variational statements that will be used in the stability/causality audits and in the derivation of Ward identities. Write the field multiplet as $S^A \in \{\Phi, \rho, \eta, T, \chi^\mu\}$ and recall the spectral derivative $D_\mu = \nabla_\mu^{(g_{\mathrm{eff}})} + (\omega_{\mu\cdot})$.

Euler–Lagrange equations (fields).

For any S^A that enters $\mathcal{L}$ only through S^A and $D_\mu S^A$ we have

$$\boxed{\mathcal{E}_A \;:=\; \frac{\partial \mathcal{L}}{\partial S^A} - D_\mu\left(\frac{\partial \mathcal{L}}{\partial (D_\mu S^A)}\right) \;=\; 0} \tag{2.12}$$

with D_μ acting adjointly on fiber indices. Using (2.3)–(2.5) this yields, schematically,

$$K_{AB}\, g_{\text{eff}}^{\mu\nu} D_\mu D_\nu S^B + (\partial_C K_{AB})\, g_{\text{eff}}^{\mu\nu} (D_\mu S^B)(D_\nu S^C) + \frac{\partial U}{\partial S^A} + \cdots \;=\; 0,$$

where the dots denote terms from the χ-vorticity block and constraints.

Connection equation (Yang–Mills form).

Varying S with respect to $\omega_\nu = \omega_\nu^A T_A$ gives

$$\boxed{D_\mu\Big(g_{AB}(\mathcal{S})\, g_{\text{eff}}^{\mu\alpha} g_{\text{eff}}^{\nu\beta}\, \mathcal{F}^B_{\alpha\beta}\Big) \;=\; J_A{}^\nu} \tag{2.13}$$

with matter current $J_A{}^\nu := -\,\partial\mathcal{L}/\partial\omega_\nu^A$ coming from the $D_\mu S^A$ terms in (2.3) (and possible minimal couplings in $D_\mu\Phi$). Equation (2.13) is covariant under $\mathrm{Iso}(\mathcal{F}, h)$ and obeys the Bianchi identity $D_{[\mu}\mathcal{F}_{\alpha\beta]} = 0$.

Metric variation and stress tensor.

Define the (Hilbert) stress tensor by

$$\boxed{T_{\mu\nu} \;:=\; -\frac{2}{\sqrt{-\det g^{\text{eff}}}}\,\frac{\delta S}{\delta g^{\text{eff}\,\mu\nu}} \;=\; 2\,\frac{\partial \mathcal{L}}{\partial g^{\text{eff}\,\mu\nu}} - g_{\mu\nu}^{\text{eff}}\,\mathcal{L}} \tag{2.14}$$

(where the last equality holds when $\mathcal{L}$ depends on g_{eff} but not on its derivatives). Diffeomorphism invariance gives the covariant conservation law

$$\nabla_\mu^{(g_{\text{eff}})} T^{\mu\nu} \;=\; J_A{}^\mu\, \mathcal{F}^{A\,\nu}{}_\mu + \mathcal{E}_A\, D^\nu S^A, \qquad \Rightarrow \qquad \nabla_\mu^{(g_{\text{eff}})} T^{\mu\nu} = 0 \quad \text{on shell.} \tag{2.15}$$

If desired, a Belinfante symmetrization can be performed to absorb spin currents; it leaves (2.15) intact.

Internal symmetries and Noether currents.

Let $\delta_\epsilon S^A = \epsilon^a (X_a)^A{}_B S^B$ be an infinitesimal global transformation with constant parameters ϵ^a and generators X_a. The canonical Noether current is

$$\boxed{J_a^\mu \;=\; \frac{\partial \mathcal{L}}{\partial (D_\mu S^A)}\,(X_a)^A{}_B\, S^B} \tag{2.16}$$

and obeys $\nabla_\mu^{(g_{\rm eff})} J_a^\mu = 0$ on shell. For local (gauged) symmetries, replacing ∂_μ by D_μ promotes (2.16) to the covariant Ward identity

$$\boxed{D_\mu J_A^\mu \;=\; 0 \quad \text{on shell}, \qquad J_A^\mu \equiv \frac{\partial \mathcal{L}}{\partial(D_\mu S^B)}\,(T_A)^B{}_C\, S^C,} \tag{2.17}$$

which is equivalent to (2.13) after using the Bianchi identity.

PO (hyperbolicity / energy estimate).

Linearize (2.12) about a smooth background and collect the perturbations into a vector U. The system can be cast in first-order symmetric-hyperbolic form

$$A^0(\bar{\mathcal{S}})\,\partial_t U + A^i(\bar{\mathcal{S}})\,\partial_i U = \text{lower order}, \qquad A^0 \succ 0, \quad A^i = (A^i)^\top, \tag{2.18}$$

with A^0 block-diagonal in the kinetic matrix K_{AB} and in the χ- and gauge sectors. The associated energy

$$\mathcal{E}[U](t) := \tfrac{1}{2}\int_{\Sigma_t} U^\top A^0 U \,\mathrm{d}\Sigma_t \tag{2.19}$$

satisfies $\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{E} \le C\,\mathcal{E}$ for bounded coefficients, establishing well-posedness on the declared DoV. In parallel, forward-limit positivity for low-energy EFT blocks (constructed from (2.8) and the quadratic sector of (2.5)) ensures the absence of ghostly dispersions and fixes the signs of Wilson coefficients.

AR.

Symbolic notebooks compute $\mathcal{E}_A$, J_A^μ, and $T_{\mu\nu}$ from a user-supplied $\mathcal{L}$ template, check unit consistency, and emit JSON/HDF5 stubs used by the PDE and gate runners. Unit tests verify: (i) dimensions of each term in $\mathcal{L}$; (ii) symmetry of $T_{\mu\nu}$; and (iii) on-shell conservation (2.15).

Acceptance 2.1. S/G Cone LMIs $\mu_{\min} \ge 0$ on the declared backgrounds and directions; forward-limit positivity for the relevant EFT quadratic blocks (dispersion matrix) is semidefinite nonnegative. Certificates and unit audits are shipped with the notebooks.

Chapter 3

Ehresmann→Yang–Mills; Gravity Coupling

Prelude. The internal fiber $(\mathcal{F}, h)$ carries a metric h and an isometry group $G := \mathrm{Iso}(\mathcal{F}, h)$. As the spectral descriptors $\mathcal{S} = \{\Phi, \rho, \eta, \chi^\mu, T\}$ vary over spacetime, so do the local fiber frames; the smooth gluing of these frames over M canonically equips the theory with a principal G–bundle and an Ehresmann connection. In this chapter we formalize this geometric emergence and show how it yields a local Yang–Mills sector that couples to the emergent metric g_{eff} fixed in Chapter 2.

3.1 Principal bundle from fiber isometries

Structure group.

Let $(\mathcal{F}, h)$ be a connected Riemannian (or pseudo-Riemannian) manifold that models the internal degrees of freedom carried by the spectral fields. Its isometry group $G := \mathrm{Iso}(\mathcal{F}, h)$ is a Lie group with Lie algebra $\mathfrak{g} := \mathrm{Lie}(G)$. Physical fields S^A transform in representations R of G, with generators $T_A \in \mathfrak{g}$ satisfying $[T_A, T_B] = f_{AB}{}^C T_C$ and $T_A^\top \eta_R + \eta_R T_A = 0$ whenever R is h-orthogonal (metric compatibility).

Local frames and transition functions.

Choose on each open set $U_i \subset M$ a smooth *fiber frame* $u_i : U_i \to \mathrm{Fr}(\mathcal{F})$ (e.g. an h–orthonormal frame or a distinguished chart). On overlaps $U_i \cap U_j$ the frames are related by

$$u_i(x) = u_j(x)\, g_{ij}(x), \qquad g_{ij} : U_i \cap U_j \to G, \tag{3.1}$$

and the family $\{g_{ij}\}$ obeys the *cocycle conditions* $g_{ij} g_{jk} = g_{ik}$ on triple overlaps and $g_{ii} = \mathbb{I}$. The data (3.1) define a principal G–bundle

$$\pi : P \longrightarrow M, \qquad P \cong \bigsqcup_i (U_i \times G)/\sim, \tag{3.2}$$

with right action $R_g(p) = p \cdot g$ and local trivialisations $\psi_i : \pi^{-1}(U_i) \to U_i \times G$.

Associated bundles and matter fields.

Given a representation $R : G \to \mathrm{GL}(V_R)$, the *associated bundle* $E_R := P \times_R V_R \to M$ carries sections that transform as $\psi_i \mapsto R(g_{ij})^{-1}\psi_j$ on overlaps. The spectral multiplet S^A is precisely a collection of sections in suitable E_R's; in particular, fiber scalars correspond to the trivial representation.

Ehresmann connection (existence and properties).

An *Ehresmann connection* on P is a $\mathfrak{g}$–valued one-form $\mathcal{A} \in \Omega^1(P, \mathfrak{g})$ such that: (i) it reproduces the generators on vertical vectors, and (ii) is equivariant under the right action: $R_g^*\mathcal{A} = \mathrm{Ad}_{g^{-1}}\mathcal{A}$. Locally on U_i it is represented by a gauge potential $\mathcal{A}^{(i)} = \mathcal{A}_\mu^{(i)}\,\mathrm{d}x^\mu$ with the usual transition rule on overlaps:

$$\mathcal{A}_\mu^{(i)} \;=\; g_{ij}^{-1}\mathcal{A}_\mu^{(j)} g_{ij} + g_{ij}^{-1}\partial_\mu g_{ij}. \tag{3.3}$$

A connection compatible with h (metric preservation along horizontal transport) always exists; when the fiber frames are h-orthonormal one has the *metricity constraint* $D_\mu h = 0$, which implies in components that the connection matrices satisfy $(\mathcal{A}_\mu)^\top \eta_F + \eta_F \mathcal{A}_\mu = 0$ for the fiber metric η_F (e.g. antisymmetry in $\mathfrak{so}(n)$).

Covariant transport and minimal coupling.

For a field $\psi \in \Gamma(E_R)$, the spectral (gauge) covariant derivative is

$$D_\mu \psi := \nabla_\mu^{(g_{\mathrm{eff}})}\psi + \mathcal{A}_\mu^A (T_A)_R \psi, \tag{3.4}$$

which reduces to the Levi–Civita derivative for fiber scalars. The replacement $\partial_\mu \to D_\mu$ in the kinetic block of $\mathcal{L}$ implements minimal coupling, and the bundle metric ensures that D_μ preserves the fiber inner products used in the action.

Curvature and Bianchi (preview).

The curvature two-form $\mathcal{F} = \mathrm{d}\mathcal{A} + \mathcal{A} \wedge \mathcal{A}$ and its local components $\mathcal{F}_{\mu\nu}$ will be developed in the next subsection; here we record the structural identity

$$D\mathcal{F} \equiv \mathrm{d}\mathcal{F} + [\mathcal{A}, \mathcal{F}] = 0, \qquad \text{(Bianchi)}. \tag{3.5}$$

Equation (3.5) is the cornerstone for the Ward/BRST identities used later and for the gauge-constraint algebra in the PDE layer.

Gate anchors.

The construction above ties directly to the gate suite: (i) G: the connection is local with declared radius L_{loc} on (M, g_{eff}) and respects cone LMIs; (ii) S: metricity $D_\mu h = 0$ and the positive fiber metric in the Yang–Mills block guarantee a positive quadratic form in

the principal symbol; (iii) TQ: the bundle data provide the natural channel algebra on which CPTP conditions are certified in Part 12.

3.1.1 Connection $\mathcal{A}_\mu$, curvature $\mathcal{F}_{\mu\nu}$, gauge charges

Connection and curvature.

On the principal G–bundle $P \to M$, an Ehresmann connection is represented locally by a $\mathfrak{g}$–valued one–form $\mathcal{A} = \mathcal{A}_\mu \, dx^\mu$ with transformation

$$\mathcal{A}'_\mu = g^{-1}\mathcal{A}_\mu g + g^{-1}\partial_\mu g, \qquad g : U \to G. \tag{3.6}$$

Its curvature (field strength) is

$$\mathcal{F}_{\mu\nu} := \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu + [\mathcal{A}_\mu, \mathcal{A}_\nu], \qquad \mathcal{F}'_{\mu\nu} = g^{-1}\mathcal{F}_{\mu\nu} g, \tag{3.7}$$

and obeys the Bianchi identity $D_{[\mu}\mathcal{F}_{\nu\rho]} = 0$ with $D_\mu := \nabla_\mu^{(g^{\mathrm{eff}})} + [\mathcal{A}_\mu, \cdot]$ acting in the adjoint.

Yang–Mills block and equations of motion.

Introduce the fiber (Killing) metric $g_{AB}(\mathcal{S}) \succ 0$ on $\mathfrak{g}$ and define the constitutive tensor

$$G_A^{\ \mu\nu} := g_{AB}(\mathcal{S})\, g_{\mathrm{eff}}^{\mu\alpha} g_{\mathrm{eff}}^{\nu\beta}\, \mathcal{F}^B_{\alpha\beta}. \tag{3.8}$$

The Yang–Mills part of the Lagrangian is

$$\mathcal{L}_{\mathrm{YM}} \;=\; -\tfrac{1}{4}\, G_A^{\ \mu\nu}\, \mathcal{F}^A_{\mu\nu}. \tag{3.9}$$

Varying the total action with respect to $\mathcal{A}_\nu$ yields the gauge equations with matter current J_A^ν:

$$\boxed{D_\mu G_A^{\ \mu\nu} \;=\; J_A^{\ \nu},} \qquad J_A^{\ \nu} := -\frac{\partial \mathcal{L}}{\partial \mathcal{A}^A_\nu} \;=\; \frac{\partial \mathcal{L}}{\partial (D_\nu S^B)}\, (T_A)^B{}_C\, S^C. \tag{3.10}$$

Taking the covariant divergence and using Bianchi gives the Ward identity $D_\nu J_A^{\ \nu} = 0$ on shell.

Stress tensor (YM contribution).

The gauge-field share of the Hilbert stress tensor is

$$T^{\mathrm{YM}}_{\mu\nu} \;=\; g_{AB}\Big(\mathcal{F}^A{}_{\mu\alpha}\, \mathcal{F}^B{}_\nu{}^\alpha - \tfrac{1}{4}\, g^{\mathrm{eff}}_{\mu\nu}\, \mathcal{F}^A_{\alpha\beta}\, \mathcal{F}^{B\,\alpha\beta}\Big), \tag{3.11}$$

and is covariantly conserved together with the matter part once the field equations hold.

Gauge charges and Gauss law.

Choose a spacelike Cauchy surface Σ with future unit normal n_μ and induced volume element $\mathrm{d}\Sigma$. The *charge density* and the total charge associated with the generator T_A are

$$\rho_A := n_\nu J_A^{\ \nu}, \qquad Q_A[\Sigma] := \int_\Sigma \rho_A \, \mathrm{d}\Sigma. \tag{3.12}$$

Projecting (3.10) along n_ν gives the Gauss constraint

$$\boxed{D_i\Big(G_A^{\ i0}\Big) \ = \ \rho_A, \qquad i = 1, 2, 3,} \tag{3.13}$$

which generates gauge transformations in the Hamiltonian picture. In the absence of matter ($\rho_A = 0$) and for suitable falloff, $Q_A[\Sigma] = \oint_{\partial\Sigma} dS_{ij}\, G_A^{\ ij}$ reduces to a surface flux (Abelian case: $Q = \oint \vec{E} \cdot d\vec{S}$).

Charge algebra and first-class property.

Let $\epsilon = \epsilon^A T_A$ be a compactly supported test function. Define the smeared generator $\mathcal{Q}[\epsilon] := \int_\Sigma \epsilon^A \left(D_i G_A^{\ i0} - \rho_A\right) \mathrm{d}\Sigma$. Its Poisson bracket with fields reproduces the infinitesimal gauge variation, and $\{\mathcal{Q}[\epsilon], \mathcal{Q}[\tilde{\epsilon}]\} = \mathcal{Q}\Big[[\epsilon, \tilde{\epsilon}]\Big]$, i.e. the constraints are *first class*. Quantum consistency requires the absence (or controlled inflow cancellation) of anomalies so that $D_\mu J_A^{\ \mu} = 0$ remains valid at the operator level.

Coupling to matter and minimal examples.

For a matter field ψ in representation R,

$$\mathcal{L}_{\mathrm{kin}}[\psi] = \tfrac{1}{2}\, \eta_R\, g^{\mathrm{eff}\,\mu\nu}\, (D_\mu\psi)^\dagger (D_\nu\psi), \qquad D_\mu\psi = \nabla_\mu^{(g^{\mathrm{eff}})}\psi + \mathcal{A}_\mu^A (T_A)_R \psi, \tag{3.14}$$

which contributes to J_A^ν by $J_A^{\ \nu} = \eta_R\, \Re\Big[(D^\nu\psi)^\dagger (T_A)_R \psi\Big] + \cdots$ and to the total $T_{\mu\nu}$. In an Abelian sector ($G = \mathrm{U}(1)$), $\mathcal{A}_\mu \mapsto \mathcal{A}_\mu + \partial_\mu\alpha$, $\mathcal{F}_{\mu\nu} = \partial_{[\mu}\mathcal{A}_{\nu]}$, and (3.13) reduces to $\nabla_i E^i = \rho$ with $E^i := G^{i0}$.

Topological charges (optional, model-dependent).

If $\pi_3(G) \neq 0$, the second Chern number

$$k \ = \ \frac{1}{8\pi^2} \int_M \mathrm{Tr}(\mathcal{F} \wedge \mathcal{F}) \in \mathbb{Z} \tag{3.15}$$

is an integer that labels instanton sectors and acts as a conserved *topological charge*. In phases with an adjoint order parameter one can also define magnetic monopole charges via surface integrals of $\mathrm{Tr}(\hat{\Phi}\, \mathcal{F})$, with $\hat{\Phi}$ a unit-norm Higgs–like field.

Gate anchors.

Positivity of g_{AB} in (3.8) ensures a positive quadratic form in the principal symbol (S); locality of $\mathcal{A}_\mu$ with the declared radius and cone feasibility for g^{eff} guarantee causal

propagation (G). These statements are certified numerically in the gate runner.

3.2 Coupling to g_{eff}; Bianchi and constraints

We now couple the gauge sector to the emergent geometry g^{eff} and collect the differential identities and constraint structure implied by diffeomorphism and gauge invariance. Throughout, all covariant derivatives ∇ are Levi–Civita with respect to g^{eff}, and D_μ denotes the adjoint/gauge-covariant extension.

Metric field equation (backreaction on g^{eff}).

With the geometric block $\mathcal{L}_{\text{geom}} = \frac{1}{2\kappa_G} R(g^{\text{eff}}) + \Lambda_{\text{eff}}(\mathcal{S})$ (cf. (2.11)), variation of the full action with respect to $g^{\text{eff}\,\mu\nu}$ yields

$$\frac{1}{2\kappa_G} G_{\mu\nu}(g^{\text{eff}}) + \Theta_{\mu\nu}(\mathcal{S}; g^{\text{eff}}) \;=\; T^{\text{tot}}_{\mu\nu} \;=\; T^{\text{matter}}_{\mu\nu} + T^{\text{YM}}_{\mu\nu}, \tag{3.16}$$

where $G_{\mu\nu}$ is the Einstein tensor of g^{eff}, T^{YM} is (3.11), and T^{matter} is the Hilbert tensor from (2.14). The *improvement* $\Theta_{\mu\nu}$ collects explicit metric variations of the descriptor-dependent coefficients (e.g. $K_{AB}(\mathcal{S})$, $g_{AB}(\mathcal{S})$, $\Lambda_{\text{eff}}(\mathcal{S})$) and implicit dependence of g^{eff} on $\mathcal{S}$ via the ansatz (2.9). By construction,

$$\nabla^{(g^{\text{eff}})}_\mu \Big(\tfrac{1}{2\kappa_G} G^\mu{}_\nu + \Theta^\mu{}_\nu\Big) \;=\; 0, \qquad \Rightarrow \qquad \nabla^{(g^{\text{eff}})}_\mu T^{\text{tot}\,\mu}{}_\nu \;=\; 0 \;\; \text{(on shell)}. \tag{3.17}$$

Equation (3.17) is the covariant energy–momentum conservation law induced by the geometrical Bianchi identity $\nabla_\mu G^\mu{}_\nu = 0$.

Gauge equations and Ward identity (recall).

Varying with respect to $\mathcal{A}_\mu$ gives (3.10), $D_\mu G_A^{\ \mu\nu} = J_A^{\ \nu}$, implying the covariant Ward identity $D_\nu J_A^{\ \nu} = 0$ on shell. Together with (3.17), the pair encodes the diffeomorphism $\times$ gauge symmetry content of the theory.

Locality radius and causal cones (G).

All nonminimal couplings and instrument kernels are constrained to have support inside geodesic balls of radius L_{loc} with respect to g^{eff}:

$$\operatorname{supp} \mathcal{K}(x,y) \subset \{\, y \in M \mid d_{g^{\text{eff}}}(x,y) \le L_{\text{loc}} \,\}.$$

The *gravity cone* feasibility is certified by the LMI condition $\mu_{\min}(\mathsf{C}(n; g^{\text{eff}}, \mathcal{S})) \ge 0$ for all unit covectors n (cf. (10.9)). These two statements jointly ensure that signal and constraint propagation remain subluminal with respect to the cone of g^{eff} on the declared DoV.

Diffeomorphism constraints (Dirac algebra).

Perform a 3+1 split of (M, g^{eff}) with lapse N, shift N^i, and induced spatial metric h_{ij}. Denote canonical momenta by (h_{ij}, π^{ij}) for geometry and $(\mathcal{A}_i^A, E_A^i)$ for the gauge sector, where $E_A^i := G_A^{\ i0}$. The primary constraints are the vanishing momenta of (N, N^i); first-class secondary constraints read

$$\mathcal{H}_\perp = \frac{1}{2\kappa_G}\Big(\mathcal{R} + K^2 - K_{ij}K^{ij}\Big) + \mathcal{H}_\perp^{\text{matter}} + \frac{1}{2} g_{AB}\, h_{ij} E^{iA} E^{jB} + \frac{1}{4} g^{ij} g^{kl}\, g_{AB}\, \mathcal{F}_{ik}^A \mathcal{F}_{jl}^B \approx 0, \tag{3.18}$$

$$\mathcal{H}_i = -2\, h_{ij}\, \nabla_k \pi^{jk} + \mathcal{P}_i^{\text{matter}} + \mathcal{F}_{ij}^A E_A^j \approx 0, \tag{3.19}$$

$$\mathcal{G}_A = D_i E^i{}_A - \rho_A \approx 0, \tag{3.20}$$

with $\mathcal{R}$ the spatial Ricci scalar of h_{ij} and K_{ij} the extrinsic curvature. Smearing with test functions (N, N^i, ϵ^A) gives generators $H_\perp[N]$, $H[\vec{N}]$, and $G[\epsilon]$ obeying the (first-class) Dirac algebra

$$\{H_\perp[N], H_\perp[M]\} = H\Big[\, h^{ij}(N\partial_j M - M\partial_j N)\,\Big], \tag{3.21}$$

$$\{H[\vec{N}], H_\perp[M]\} = H_\perp[\mathcal{L}_{\vec{N}} M], \qquad \{H[\vec{N}], H[\vec{M}]\} = H[[\vec{N}, \vec{M}]], \tag{3.22}$$

$$\{G[\epsilon], G[\tilde{\epsilon}]\} = G[[\epsilon, \tilde{\epsilon}]], \qquad \{H_\perp[N], G[\epsilon]\} = 0 = \{H[\vec{N}], G[\epsilon]\}, \tag{3.23}$$

expressing closure of diffeo and gauge constraints and their mutual compatibility.[1]

Ward/BRST identities on the physical quotient.

Introduce ghosts c^A for G and $(\bar{c}_A, B_A)$ for gauge fixing, with BRST differential s defined by

$$s\,\mathcal{A}_\mu = D_\mu c, \qquad s\,c = -\tfrac{1}{2}[c, c], \qquad s\,\bar{c} = B, \qquad s\,B = 0,$$

and extend s to matter fields by their representation of $\mathfrak{g}$. For a gauge-fixing fermion Ψ one defines $\mathcal{L}_{\text{gf+gh}} = s\,\Psi$ and the gauge-fixed action $S_{\text{gf}} = S + \int \sqrt{-g^{\text{eff}}}\, \mathcal{L}_{\text{gf+gh}}$. Nilpotency $s^2 = 0$ implies

$$s\, S_{\text{gf}} = 0 \quad \Rightarrow \quad \langle s\,\mathcal{O}\rangle_{S_{\text{gf}}} = 0 \ \text{ for BRST-closed observables } \mathcal{O}, \tag{3.24}$$

and the *physical algebra* is the cohomology $H^0(s)$ at ghost number 0. On curved background (M, g^{eff}) the BRST Ward identity reduces in the 1PI functional to $D_\mu \langle J_A^\mu \rangle = 0$; anomalies, if present, must either cancel (via spectrum/charge assignments, cf. V7–SM mapping) or be accounted for by an inflow mechanism so that the physical quotient remains well-defined.

[1] Possible improvement terms from $\Theta_{\mu\nu}$ do not spoil first-classness on the DoV backgrounds considered here; they enter $\mathcal{H}_\perp^{\text{matter}}$ and $\mathcal{P}_i^{\text{matter}}$ in a covariant way.

Hadamard prerequisites (S).

Quantization requires that the linearized Klein–Gordon/Dirac-type operators be *Green-hyperbolic* on (M, g^{eff}) (global hyperbolicity of g^{eff} on the DoV), and that the chosen state be *Hadamard*. In microlocal form,

$$\mathrm{WF}(\omega_2) = \left\{ (x,k;x',-k') \in T^*M^2 \setminus \{0\} \;\middle|\; (x,k) \sim (x',k'),\ k \in \mathcal{N}^+_{g^{\text{eff}}} \right\},$$

ensuring positivity of the CCR/CAR and stability of renormalized composite operators. These conditions, together with cone feasibility and locality radius L_{loc}, constitute the prerequisites audited under S/G.

Acceptance 3.1. G (locality) / S (Hadamard prerequisites) Locality: all nonminimal couplings and instrument kernels have support within $B_{g^{\text{eff}}}(x, L_{\text{loc}})$ and pass cone LMIs $\mu_{\min} \geq 0$ on the declared backgrounds. Quantization: linearized operators are Green-hyperbolic on the DoV; reference states satisfy the Hadamard microlocal spectrum condition; BRST Ward identities hold (no uncancelled anomalies). Certificates and numerical margins are shipped with the gate runner.

Chapter 4

HDF5 v7 — Operational Data Schema

4.1 Purpose and Scope

This chapter specifies the `HDF5` layout (version `v7`) used by the USFT toolchain. Goals: one–click reproducibility, automated `S/G/TQ/D` audits, and a tight link from equations to predictions/proofs in a single, unit–annotated, constraint–checked file.

4.2 Top-Level Attributes (Root /)

The root `/` *must* contain:

- `schema_version` = `"v7"` (string, required);
- `build_time_utc` = `YYYY-MM-DDThh:mm:ssZ` (ISO–8601);
- `git_commit`, `env_lock` (hash/environment string);
- `generator` = `"usft-v7-pipeline"` (or equivalent);
- `units_system` = `"SI"` or `"natural(c=hbar=1)"`.

4.3 Global Conventions

- **Units:** every numeric dataset/group carries a `units` attribute (string).
- **Axes/dimensions:** every numeric dataset carries an `axes` attribute (ordered list of axis names) and links to coordinate arrays under `/coords`.
- **Types:** floats `float64`; indices `int32/int64`; booleans `uint8` (0/1).
- **Provenance:** each top–level group includes `hash_sha256` covering its inputs.

4.4 Core Groups at Root

The following groups are mandatory under `/`. Tables below list required keys.

4.4.1 `/msp/inputs`

Key	Type	Units	Description
`kappa_G`	float64	(per system)	Geometric/spectral normalization constant.
`N_u`, `N_d`, `N_l`	int32	–	Family counts (up/down/leptons).
`topology_indices`	int32[n]	–	Topological indices (winding numbers, bundle ranks, ...).

4.4.2 `/action/terms`

Key	Type	Units	Description
`kinetic`	float64[...]	energy density	Kinetic/metric blocks (e.g. K, Z tensors).
`potential`	float64[...]	energy density	$V(\Phi, \rho, \eta, T, \chi)$.
`mixings`	float64[...]	energy density	Cross–couplings among descriptors.
`rayleigh`	float64[...]	power density	Rayleigh dissipation functional (Gate D).

Note: every array carries an `axes` attribute (e.g. `"[a,k,theta]"`).

`/action/linearization`

Key	Type	Units	Axes	Description
`K`	`float64[...]`	–	[a,k,theta]	Perturbation kinetic matrix (ghost-free test).
`Z`	`float64[...]`	–	[a,k,theta]	Symplectic/flux matrix (cones, group velocity).
`M2`	`float64[...]`	energy^2	[a,k,theta]	Mass/coupling blocks; gap maps and thresholds.

Key	Type	Units	Axes	Description
flows.mu	float64[n_mu]	energy	[mu]	RG scale grid.
flows.beta	float64[n_mu,n_g]	–	[mu,g]	Beta functions for g_i.
flows.g123	float64[n_mu,3]	–	[mu,i]	Tracks (g_1, g_2, g_3).
scheme	struct/string	–	–	Scheme name + loop order (e.g., $\overline{\text{MS}}$/MOM).
thresholds	float64[...]	energy	[species]	Decoupling masses/scales + validity flags.

Key	Type	Units	Axes	Description
K	float64[3,3]	–	[I,J]	3×3 kinetic Gram from YM anchor.
O	float64[3,3]	–	[I,J]	Orthogonal diagonalizer of K (mixing removal).
alpha_tilde	float64[3]	–	[I]	Diagonal entries after $K = O^\top \mathrm{diag}(\alpha)\, O$.
nY_convention	string	–	–	{SM, GUT(5/3)}.
g123_ref	float64[3]	–	[i]	(g_1, g_2, g_3) at μ_{ref}.
thetaW_ref	float64	–	–	$\sin^2\theta_W(\mu_{\text{ref}})$.
charges	table	–	–	Matter reps $(\text{rep}, Y, SU2, SU3, \text{mult})$.
anomalies	table	–	–	Residuals for $[SU(3)]^2U(1)$, $[SU(2)]^2U(1)$, $[U(1)]^3$, grav^2–$U(1)$.
status	string	–	–	{pass|warn|fail} for SM matching.

/rg

/sm_map

/sm_map/charges

- `table` (compound): columns `name:string`, `R3:string`, `R2:string`, `Y:float64`, `mult:uint32`.
- `indices` (group): optional scalars for T and d if nonstandard reps are used.

/sm_map/anomalies

- `residuals` float64[4] (axes=[33Y,22Y,YYY,ggY]).
- `norm_score` float64 (ρ as defined above).
- `eps_A` float64 (tolerance used).
- `pass` uint8 (1=pass, 0=fail).

4.4.3 /backgrounds

Key	Type	Units	Description
`minkowski`	group	–	Flat background sheets/parameters.
`frw`	group	–	Tables for $H(a)$, $w_{\text{eff}}(a)$, g_{eff}.
`bh_stationary`	group	–	Stationary black–hole background parameters.

Inside each background: `params` (scalars), `profiles` (arrays), `cones` (LMI margins/front speeds); each with `units`/`axes`.

4.4.4 /cones/lmi

Key	Type	Units	Description
`mu_min`	float64[...]	–	Minimum eigenvalue over LMI grids.
`L_loc`	float64[...]	length	Locality/causality radius.
`v_front`	float64[...]	speed	Front speeds vs. c_{eff}.

4.4.5 /quantum/gksl

Key	Type	Units	Description
`choi_eigs`	float64[n]	–	Choi eigen–spectrum (CPTP margins).
`choi_traces`	float64[n]	–	Trace/contractivity diagnostics.
`di_bounds`	float64[m]	–	Device–independent no–signalling bounds.

4.4.6 /rg/flows

Key	Type	Units	Description
`mu`	float64[n_μ]	energy	RG scale grid.
`beta`	float64[n_μ, n_g]	–	Beta functions for couplings g_i.
`g123`	float64[$n_\mu, 3$]	–	Tracks for g_1, g_2, g_3.

4.4.7 /predictions

Key	Type	Units	Description
`sin2thetaW_MZ`	float64	–	Predicted $\sin^2\theta_W$ at M_Z.
`STU`	float64[3]	–	Oblique parameters (S, T, U).
`cT`	float64[n_f]	–	GW speed $c_T(f)$.
`weff`	float64[n_a]	–	$w_{\text{eff}}(a)$ from η–relaxation.
`g_star`	float64[n_k]	–	Statistical anisotropy amplitude.

4.4.8 /windows/kill

Key	Type	Units	Description
`obs_id`	string	–	Observable (e.g. `"cT"`, `"sin2thetaW"`).
`bound_type`	string	–	`"two-sided"` or `"one-sided"`.
`lower`, `upper`	float64	[obs units]	Decision window bounds.
`confidence`	float64	–	Confidence (e.g. 0.95).

4.4.9 /audits

Key	Type	Units	Description
gate_S, gate_G, gate_TQ, gate_D	uint8	–	1/0 pass indicators; details in attributes.
dov_tiling	float64[...]	–	DoV tiling and boundary margins.

4.4.10 /coords

Shared coordinate arrays (mandatory if axes is used):

Key	Type	Units	Description
a	float64[n_a]	–	Scale factor.
k	float64[n_k]	1/length	Wavenumber.
theta	float64[n_θ]	rad	Angle w.r.t. $\hat{\chi}$.
f	float64[n_f]	Hz	GW frequency.
mu	float64[n_μ]	energy	RG scale.

4.4.11 /runs

Run metadata for reproducibility:

- solver (string), grid (YAML embedded as string);
- cfl, tol_abs, tol_rel (float64);
- seed (int64), elapsed_sec (float64).

4.5 Example Tree Layout

```
/
 |- msp/
 |  |- inputs/{kappa_G, N_u, N_d, N_l, topology_indices}
 |- action/
 |  |- terms/{kinetic, potential, mixings, rayleigh}
 |- backgrounds/{minkowski, frw, bh_stationary}/{params, profiles, cones}
 |- cones/
 |  |- lmi/{mu_min, L_loc, v_front}
 |- quantum/
 |  |- gksl/{choi_eigs, choi_traces, di_bounds}
 |- rg/flows/{mu, beta, g123}
```

```
|- predictions/{sin2thetaW_MZ, STU, cT, weff, g_star}
|- windows/kill/{obs_id, bound_type, lower, upper, confidence}
|- audits/{gate_S, gate_G, gate_TQ, gate_D, dov_tiling}
|- coords/{a, k, theta, f, mu}
`- runs/{solver, grid, cfl, tol_abs, tol_rel, seed, elapsed_sec}
```

4.6 Integrity Constraints

1. **Units required:** every numeric dataset has `units` and `axes`.
2. **Valid axes:** each axis named in `axes` exists as `/coords/<name>`.
3. **Gate G:** if `/cones/lmi/v_front` exists, enforce $\max(v_{\text{front}}/c_{\text{eff}}) \leq 1 + \varepsilon_G$ with attributes `c_eff`, `eps_G`.
4. **CPTP:** $\min(\texttt{choi_eigs}) \geq -\varepsilon_{TQ}$.
5. **Kill windows:** each record in `/windows/kill` satisfies `lower` $\leq$ `upper` and `obs_id` matches a path in `/predictions`.
6. **DoV:** `/audits/dov_tiling` exists and lists all triggers (gap–closure, CP–loss, cone–violation).

Acceptance 4.1. Schema acceptance An `.h5` is *valid* when (i) root attributes `schema_version, build_time_utc, units_system, git_commit` exist; (ii) integrity constraints 1–6 pass; and (iii) the S/G/TQ/D flags under `/audits` indicate pass, with margins documented in group attributes.

Chapter 5

YM Anchor: From the Lie–Metric to SM Gauge Couplings

5.1 Spectral YM block and projections

Let $(\mathfrak{g}, h_{AB})$ be the internal Lie algebra equipped with a positive–definite *Lie–metric* h_{AB}. The spectral Yang–Mills term reads

$$\mathcal{L}_{\rm YM} \;=\; -\frac{\kappa_G}{4}\, h_{AB}\, \mathcal{F}^A_{\mu\nu}\, \mathcal{F}^{B\,\mu\nu}, \qquad \mathcal{F}^A_{\mu\nu} = \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\mu + f^A{}_{BC}\, \mathcal{A}^B_\mu \mathcal{A}^C_\nu. \tag{5.1}$$

An embedding of $G_{\rm SM} = U(1)_Y \times SU(2)_L \times SU(3)_c$ is specified by projectors (embedding tensors)

$$E^A{}_{(1)} \quad (Y), \qquad E^A{}_{(2)i}\ (i = 1, 2, 3), \qquad E^A{}_{(3)a}\ (a = 1, \ldots, 8),$$

so that

$$B_\mu = E^A{}_{(1)}\, \mathcal{A}^A_\mu, \quad W^i_\mu = E^A{}_{(2)i}\, \mathcal{A}^A_\mu, \quad G^a_\mu = E^A{}_{(3)a}\, \mathcal{A}^A_\mu.$$

Diagonal (orthonormal) case.

Projecting $\mathcal{L}_{\rm YM}$ onto the SM factors yields

$$\mathcal{L}^{\rm SM}_{\rm kin} = -\frac{1}{4}\Big(\alpha_1\, B_{\mu\nu}B^{\mu\nu} + \alpha_2\, W^i_{\mu\nu}W^{i\,\mu\nu} + \alpha_3\, G^a_{\mu\nu}G^{a\,\mu\nu}\Big), \qquad \alpha_i := \kappa_G\, \Xi_i, \tag{5.2}$$

with anchor coefficients

$$\Xi_1 = h_{AB}E^A{}_{(1)}E^B{}_{(1)}, \quad \Xi_2 = h_{AB}E^A{}_{(2)i}E^B{}_{(2)i}, \quad \Xi_3 = h_{AB}E^A{}_{(3)a}E^B{}_{(3)a}.$$

Canonical rescaling gives

$$\boxed{g_1^{-2} = \alpha_1, \quad g_2^{-2} = \alpha_2, \quad g_3^{-2} = \alpha_3} \quad \Longleftrightarrow \quad \boxed{g_i^2 = \frac{1}{\kappa_G\, \Xi_i}}. \tag{5.3}$$

5.2 Kinetic Gram matrix and canonicalization (general case)

Define

$$K_{IJ} \;:=\; \kappa_G\, h_{AB}\, E^A{}_{(I)}\, E^B{}_{(J)}, \qquad K = K^\top \succ 0, \quad I, J \in \{1,2,3\}.$$

Orthogonally diagonalize

$$K \;=\; O^\top \operatorname{diag}(\tilde\alpha_1, \tilde\alpha_2, \tilde\alpha_3)\, O, \qquad O \in \mathrm{SO}(3).$$

In the rotated fields $F' := O\,F$ we have

$$\mathcal{L}^{\mathrm{SM}}_{\mathrm{kin}} \;=\; -\frac{1}{4}\sum_{I=1}^{3} \tilde\alpha_I\, F'^{I}_{\mu\nu} F'^{I\,\mu\nu},$$

so the canonically normalized couplings are

$$\boxed{g_I^{-2} = \tilde\alpha_I} \quad \text{(no mixing in the } F' \text{ basis).}$$

Equivalently, with a symmetric factor S such that $K = S^\top S$, one may write $K = (O^\top D^{1/2})(D^{1/2} O)$ with $D = \operatorname{diag}(\tilde\alpha_I)$; for auditability, store either (O, D) (spectral diagonalization) or S (Cholesky).

Remark (non-abelian blocks).

In the SM with a single $U(1)_Y$ and two simple non-abelian factors, gauge invariance implies that any kinetic mixing is confined to the abelian sector. Practically, K is block-diagonal and any orthogonal rotation O must act only within the abelian $U(1)$ subspace (or among multiple $U(1)$'s if present), while the $SU(2)$ and $SU(3)$ blocks remain unrotated up to internal orthonormal changes that preserve their Lie brackets.

5.3 Generator normalization and $U(1)_Y$ convention

For $SU(N)$ we use $\mathrm{Tr}(T^a T^b) = \frac{1}{2}\,\delta^{ab}$. For $U(1)_Y$ introduce a normalization n_Y:

$$Y_{\mathrm{canon}} = n_Y\, Y, \qquad g_1 \equiv g_Y / n_Y.$$

Typical choices: $n_Y = 1$ (SM) or $n_Y = \sqrt{5/3}$ (GUT). The same n_Y must be used in the computation of Ξ_1 and in reporting g_1.

5.4 Matter reps and anomaly worksheet

Input.

List *left-handed Weyl* fields only. For each species f, give $R_3^f \in \{1, \mathbf{3}, \overline{\mathbf{3}}, \text{adj}\}$, $R_2^f \in \{1, \mathbf{2}, \text{adj}\}$, hypercharge Y_f (use the declared n_Y so $Y_{\text{canon}} = n_Y Y$), multiplicity $m_f \in \mathbb{N}$.

Group data.

For $SU(N)$ use $T(\mathbf{N}) = \frac{1}{2}$, $T(\text{adj}) = N$, $d(\mathbf{N}) = N$, $d(\text{adj}) = N^2 - 1$.

Anomaly coefficients (SM blocks).

$$A_{33Y} = \sum_f m_f \, T_3(R_3^f) \, d_2(R_2^f) \, Y_f, \tag{5.4}$$

$$A_{22Y} = \sum_f m_f \, T_2(R_2^f) \, d_3(R_3^f) \, Y_f, \tag{5.5}$$

$$A_{YYY} = \sum_f m_f \, d_3(R_3^f) \, d_2(R_2^f) \, Y_f^3, \tag{5.6}$$

$$A_{ggY} = \sum_f m_f \, d_3(R_3^f) \, d_2(R_2^f) \, Y_f \qquad (\text{grav}^2 - U(1)). \tag{5.7}$$

Acceptance (anomaly gate).

Define the residual vector $\vec{r} = (A_{33Y}, A_{22Y}, A_{YYY}, A_{ggY})$ and the normalized score $\rho := \max_i |r_i| / \max(1, \max_f m_f)$. Pass if $\|\vec{r}\|_\infty \le \varepsilon_A$ *and* $\rho \le \varepsilon_A$. Store ε_A and the n_Y choice.

Auditability.

Save the table of reps f and the computed $(A_{33Y}, A_{22Y}, A_{YYY}, A_{ggY})$ under `/sm_map`; the replay must reproduce them.

5.5 Predictive relations at the reference scale

Fix a reference scale μ_{ref}. From (**??**) (or (5.3) in the diagonal case) obtain $g_i(\mu_{\text{ref}})$ and

$$\sin^2\theta_W(\mu_{\text{ref}}) = \frac{g_1^2}{g_1^2 + g_2^2}, \qquad \Delta_{ij}(\mu_{\text{ref}}) = \left| g_i^{-2} - g_j^{-2} \right|.$$

Transport to M_Z using the documented RG scheme (stored under `/rg/flows`).

5.6 SM covariant derivative and consistency

$$D_\mu \;=\; \partial_\mu + i\, g_1\, Y_{\text{canon}}\, B_\mu + i\, g_2\, T^i\, W_\mu^i + i\, g_3\, T^a\, G_\mu^a,$$

acting on the matter reps carried by the spectral fiber.

5.7 Acceptance checks (YM Anchor)

Checklist 5.1: YM acceptance

- **Normalization invariance.** Values of (g_1, g_2, g_3) extracted from (**??**) are invariant under orthonormal basis changes in $\mathfrak{g}$.
- **Positivity.** h_{AB} restricted to each embedded subalgebra is positive–definite; hence $K \succ 0$ and $g_i \in \mathbb{R}$.
- **Hypercharge convention.** A single n_Y is used consistently in generators, Ξ_1, and in reporting g_1; this choice is recorded.
- **Anomalies.** $[SU(3)]^2U(1)$, $[SU(2)]^2U(1)$, $[U(1)]^3$ and grav^2–$U(1)$ anomalies cancel for the embedded spectrum.
- **RG continuity.** (g_1, g_2, g_3) at μ_{ref} flow to M_Z without pathologies; the scheme/thresholds are logged.
- **Kinetic conditioning (new).** $\lambda_{\min}(K) > 0$ and $\text{cond}(K) \leq \kappa_{\max}$ on the declared DoV.
- **Record canonicalization (new).** The matrices K and S (with $K = S^\top S$) and the n_Y choice are stored under `/sm_map`, and reproduce (g_1, g_2, g_3) on replay.

5.8 HDF5 keys for the YM anchor (v7)

Store under `/sm_map`:

- `lie_metric`: h_{AB} (float64[dim,dim], units: –).
- `embeddings`: $E^A{}_{(1)}, E^A{}_{(2)i}, E^A{}_{(3)a}$ (datasets).
- `K`: the 3×3 kinetic Gram matrix (**??**).
- `S`: a canonicalization factor with $K = S^\top S$ (e.g. Cholesky).
- `alpha`: $(\alpha_1, \alpha_2, \alpha_3)$ from (5.2) (if K is diagonal).
- `g123_ref`: (g_1, g_2, g_3) at μ_{ref}.
- `nY_convention`: string {`SM`, `GUT(5/3)`}.
- `thetaW_ref`: $\sin^2\theta_W(\mu_{\text{ref}})$.
- `O`: the 3×3 orthogonal matrix from the spectral diagonalization of K.
- `D`: the diagonal eigenvalue matrix $\text{diag}(\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3)$.

All arrays carry `units/axes`; audits appear in `/audits/gate_S` and `/audits/gate_G`.

Acceptance 5.1. YM Anchor Passes when the checklist **??** is satisfied on the declared DoVand the HDF5 group `/sm_map` contains `K`, `S`, the n_Y convention, and reproduces (g_1, g_2, g_3) and $\sin^2\theta_W$ on replay.

One-line takeaway.

Geometry fixes the SM couplings: given (h_{AB}, E) at μ_{ref}, build K by (**??**), factor $K = S^\top S$, and read $g_I^{-2} = (SS^\top)_{II}$. These flow by RG to measurement scales and feed the predictive layer.

Part III

Computational Stack and Numerical Solutions

Chapter 6

Well-Posed PDE Formulation and Nondimensionalization

Purpose. This chapter turns the Unified Spectral Field Theory (USFT) into a well-posed initial–boundary value problem (IBVP) suitable for robust numerics on the declared DoV. We (i) isolate dynamical vs. constrained variables, (ii) choose gauges and auxiliary variables so that the evolution system attains a *symmetrizable hyperbolic* form with a positive energy estimate, and (iii) introduce a *nondimensionalization* map using reference scales (M_*, L_*, T_*) so that all PDE blocks are written in unit-consistent, solver-ready form.

Scope. Starting from the spectral action and its Euler–Lagrange equations, we assemble the coupled PDE system for the background classes used in Volume VII (e.g. Minkowski, FRW, stationary cones). The principal symbol and characteristic structure are exposed to certify: (1) real, bounded characteristic speeds within the effective cone (Gate G), (2) positivity of the kinetic metric and energy functional (Gate S), and (3) compatibility of Rayleigh/dissipative channels with nonnegative entropy production (Gate D). The resulting nondimensional system becomes the single source of truth for the solvers and for the Numerics Gate (§7).

Outcomes. By the end of the chapter we will have:

- a symmetrizable-hyperbolic evolution system $\partial_t U = \sum_i A^i(U)\,\partial_i U + S(U)$, with symmetric energy multiplier and cone LMIs to certify $v_{\text{front}} \leq \text{cEff}$;
- a consistent split of algebraic/elliptic *constraints* and their propagation equations, together with penalty/SAT boundary prescriptions that preserve well-posedness;
- a nondimensional variable set $\widehat{U}$ and parameters $\widehat{\Pi}$, obtained by explicit scaling rules tied to (M_*, L_*, T_*), ensuring that all terms are numerically balanced and auditably unit-free.

Interface to later chapters. The PDE form and the scaling maps defined here feed directly into: (i) the NGate acceptance tests (CFL, convergence, causality), (ii)

the Operational Background Solvers, and (iii) the Linear-Mode/Dispersion and Soliton pipelines. All numerical artifacts emitted thereafter carry the same nondimensional conventions and cite this chapter's symbol tables and scaling definitions.

6.1 Evolution equations, gauges, constraints

First–order reduction and state vector

Let $U(t, \mathbf{x})$ collect all dynamical fields in first–order form,

$$U \;=\; \Big(\Phi,\; \Pi_\Phi,\; \Psi_{i,\Phi};\; \rho,\; \Pi_\rho,\; \Psi_{i,\rho};\; \eta,\; \Pi_\eta,\; \Psi_{i,\eta};\; T,\; \Pi_T,\; \Psi_{i,T};\; \mathcal{A}_i^A,\; E_i^A\Big)^\top,$$

where $\Pi_\bullet$ are canonical time–derivatives and $\Psi_{i,\bullet} := \nabla_i^{\text{eff}}(\bullet)$ use the effective connection of (M, g_{eff}). For the gauge sector we adopt electric–magnetic variables $E_i^A := \mathcal{F}_{0i}^A$ and $B^{A\,i} := \frac{1}{2}\epsilon^{ijk}\mathcal{F}_{jk}^A$. On a declared DoV slice we write the evolution system in quasi–linear, first–order, flux–source form

$$\partial_t U \;+\; \sum_{i=1}^{3} A^i(U; g_{\text{eff}})\,\partial_i U \;=\; S(U; g_{\text{eff}}) \;+\; R(U; g_{\text{eff}}), \tag{6.1}$$

with R collecting lower–order Rayleigh/dissipative terms (Gate D). The principal symbol in direction $n \in \mathbb{S}^2$ is $\mathsf{A}(n) := n_i A^i$, with characteristic speeds $\lambda_\alpha(n)$ defined by the eigenproblem $\mathsf{A}(n)\,v_\alpha = \lambda_\alpha v_\alpha$.

Gauge choices (hyperbolic gauges)

We employ gauge conditions that yield a symmetric–hyperbolic principal part and damped constraint propagation:

Yang–Mills (generalized Lorenz): $$\mathcal{G}^A \;:=\; \nabla^\mu_{\text{eff}}\mathcal{A}^A_\mu \;+\; \zeta\,\phi^A \;=\; 0, \qquad \partial_t\mathcal{G}^A \;+\; \gamma_G\,\mathcal{G}^A \;=\; 0, \tag{6.2}$$

Abelian $U(1)$ limit: $$\partial^\mu_{\text{eff}}A_\mu + \zeta\,\varphi = 0, \quad \partial_t(\partial^\mu_{\text{eff}}A_\mu) + \gamma_G(\partial^\mu_{\text{eff}}A_\mu) = 0. \tag{6.3}$$

Here ζ is a gauge parameter and $\gamma_G > 0$ a damping rate. The auxiliary ϕ^A (resp. φ) converts the gauge condition into a first–order equation. For matter scalars/tensors we use first–order reductions with algebraic gauges for T as needed (e.g. trace or transverse conditions on specific backgrounds).

Constraint set and propagation

The full constraint vector is

$$C(U) \;=\; \Big(\mathcal{G}^A,\; \mathcal{B}^A,\; \mathcal{C}^A_{\text{Gauss}},\; C_{\text{BRST}},\; C_{\text{Noether}},\; C_{\text{Bianchi}}\Big)^\top,$$

with $\mathcal{B}^A := \nabla^{\text{eff}}_i B^{A\,i}$, $\mathcal{C}^A_{\text{Gauss}} := D^{\text{YM}}_i E^{A\,i} - \rho^A_{\text{gauge}}$, and C_{BRST} denoting the algebraic constraints on the physical quotient (ghost #0 sector). Linearizing around a smooth background yields a closed, damped, symmetric–hyperbolic subsystem

$$\partial_t C \;+\; \sum_{i=1}^{3} \mathcal{B}^i\,\partial_i C \;=\; -\Gamma_C\, C, \qquad \Gamma_C \succeq \gamma_C\,\mathbb{I},\;\; \gamma_C > 0, \tag{6.4}$$

so that $\|C(t)\|_{L^2} \le e^{-\gamma_C t}\|C(0)\|_{L^2}$ in the absence of inflow.

Symmetrizer and energy estimate

There exists a positive–definite symmetrizer $H(U; g_{\text{eff}}) \succ 0$ such that $HA^i = (HA^i)^\top$ for $i = 1, 2, 3$. Define the energy

$$\mathcal{E}[U](t) \;=\; \frac{1}{2}\int_\Omega U^\top H(U; g_{\text{eff}})\, U\; \sqrt{|\det g_{\text{eff}}|}\, d^3x. \tag{6.5}$$

Ignoring boundaries,

$$\frac{d}{dt}\mathcal{E}[U] \;=\; \int_\Omega U^\top H\, R\, dV_{\text{eff}} \;+\; \text{(lower order)}. \tag{6.6}$$

For Rayleigh terms of the form $R = -\mathsf{M}(U; g_{\text{eff}})\, U$ with $\mathsf{M} \succeq 0$, we obtain the dissipation inequality

$$\frac{d}{dt}\mathcal{E}[U] \;\le\; -\int_\Omega U^\top H\, \mathsf{M}\, U\; dV_{\text{eff}} \;+\; \text{boundary flux}. \tag{6.7}$$

Characteristic cones and Gate G

Let n be the outward unit normal to a boundary patch. The boundary matrix is $A_n := n_i A^i$ with symmetrized form $A_n^{\text{sym}} := \frac{1}{2}(HA_n + A_n^\top H)$. The local characteristic speeds are the eigenvalues of A_n, and the *effective cone* constraint reads

$$\max_\alpha \frac{|\lambda_\alpha(n)|}{\text{cEff}} \;\le\; 1 \quad \text{on the declared DoV}, \tag{6.8}$$

with cEff computed from g_{eff}. In practice we enforce $\mu_{\min} \ge 0$ on the cone–LMI built from A_n^{sym} (see the Gate Grunners).

Boundary conditions (SAT/penalty inflow)

On each boundary patch we split the characteristic fields into inflow/outflow via A_n and impose weak (penalty) conditions on the inflow part U_-:

$$\partial_t U \;+\; \sum_i A^i \partial_i U \;=\; S + R \;+\; \tau\, H^{-1} B^\top \Big(BU - g\Big)\, \delta_{\partial\Omega}, \tag{6.9}$$

where B selects the constrained combinations and g encodes the physical boundary data. Choosing $\tau \geq \tau_{\min} > 0$ yields a negative–semidefinite boundary contribution in (6.7) and preserves well–posedness (see NGate NG1/NG2).

Matter and YM blocks (principal parts)

As illustration, on a fixed (M, g_{eff}) background:

$$\textbf{Scalar } \phi: \quad \partial_t \Pi_\phi - \nabla_i^{\text{eff}} \Psi_\phi^i \;=\; \cdots, \qquad \partial_t \Psi_{i,\phi} - \nabla_i^{\text{eff}} \Pi_\phi \;=\; \cdots, \tag{6.10}$$

$$\textbf{Yang–Mills } (\mathcal{A}_i^A, E_i^A): \quad \partial_t \mathcal{A}_i^A + \alpha^j{}_i\, \partial_j \mathcal{A}_0^A - E_i^A \;=\; \cdots, \qquad \partial_t E_i^A - \epsilon_{ijk} \nabla_j^{\text{eff}} B^{Ak} \;=\; \cdots, \tag{6.11}$$

where $\alpha^j{}_i$ encodes the $(3+1)$ split of g_{eff}. Couplings to $(\rho, \eta, T, \chi^\mu)$ appear in lower–order terms (sources/mass–mixing), leaving the principal part hyperbolic on the DoV.

Well–posedness theorem (IBVP, schematic)

Theorem 6.1 (Symmetrizable–hyperbolic well–posedness)**.** *Assume: (i) smooth* $(g_{\text{eff}}$, *coefficients) on the DoV, (ii)* $H \succeq h_0 \mathbb{I}$ *and* $HA^i = (HA^i)^\top$, *(iii) cone feasibility* (6.8)*, (iv) constraint propagation* (6.4) *with* $\gamma_C > 0$, *(v) SAT penalties* (6.9) *with* $\tau \geq \tau_{\min}$, *and (vi)* $\mathsf{M} \succeq 0$. *Then the IBVP for* (6.1) *is strongly well–posed in* L^2*: there exists a unique solution with continuous dependence on the data and*

$$\|U(t)\|_{L^2_H}^2 \;+\; \int_0^t \!\!\int_\Omega U^\top H\, \mathsf{M}\, U \; dV_{\text{eff}}\, ds \;\leq\; C\Big(\|U(0)\|_{L^2_H}^2 + \|g\|_{L^2([0,t]\times\partial\Omega)}^2\Big).$$

Acceptance hooks (to NGate and Gates S/G/D)

- **Gate S:** certify $H \succeq h_0 \mathbb{I}$ and compute the energy estimate; $\log \lambda_{\min}(H)$ over the grid.
- **Gate G:** record front–speed ratios $\max_\alpha |\lambda_\alpha|/\text{cEff}$ and the minimal cone–LMI eigenvalue $\mu_{\min}$.
- **Gate D:** expose $\mathsf{M} \succeq 0$ and the discrete ledger (energy/entropy balance) used later by NG2.

- **NGate links:** NG1 (CFL bound from $\lambda_{\max}$), NG5 (constraint norms from (6.4)), NG6 (front speeds vs. cEff).

Notes on backgrounds and gauges

For FRW backgrounds we use comoving coordinates with lapse $N(a)$ and scale factor $a(t)$ from `/backgrounds/frw/params`; for stationary cones we align n with the principal spatial directions of g_{eff}. Gauge parameters (ζ, γ_G) and SAT penalties τ are included in the HDF5 manifests (`/runs/solver`) and are part of the reproducibility bundle.

6.2 Dissipation functional (Rayleigh) and Denforcement

Rayleigh structure and thermodynamic sign

On the declared DoV we model irreversible channels by a *Rayleigh dissipation density* $\mathfrak{R}(U; \nabla U)$ and the corresponding bulk functional

$$\mathcal{R}[U] \;=\; \int_\Omega \mathfrak{R}(U; \nabla U)\, dV_{\text{eff}}, \qquad dV_{\text{eff}} := \sqrt{|\det g_{\text{eff}}|}\, d^3x.$$

We assume a *quadratic, positive* form in generalized currents $J = J(U; \nabla U)$:

$$\mathfrak{R} \;=\; \tfrac{1}{2}\, J^\top \Gamma(U; g_{\text{eff}})\, J, \qquad \Gamma = \Gamma^\top \succeq 0, \tag{6.12}$$

with examples

- **Diffusion/viscosity:** $J = \nabla\Phi$, $\Gamma = \mathsf{D} \succeq 0$;
- **Color conductivity:** $J_i^A = E_i^A$ (or covariant versions), $\Gamma^{Ai,Bj} = \sigma^{AB}\delta_i{}^j$, $\sigma \succeq 0$;
- **Bulk/friction:** $J = U$, $\Gamma = \mathsf{M} \succeq 0$ (as in (6.7)).

The dissipative source in (6.1) is then $R = -\frac{\delta\mathcal{R}}{\delta U}$ (or its first–order reduction), guaranteeing nonnegative entropy production.

Energy/heat ledger and entropy production

Let $\mathcal{E}[U]$ be the symmetrized energy in §6.1. Including boundary work and sources, the balance identity reads

$$\frac{d}{dt}\,\mathcal{E}[U] \;=\; P_{\text{in}} - P_{\text{out}} \;-\; \mathcal{D}[U], \qquad \mathcal{D}[U] \;:=\; \int_\Omega J^\top \Gamma J\, dV_{\text{eff}} \;\geq\; 0, \tag{6.13}$$

with $P_{\text{in/out}}$ the power fluxes through $\partial\Omega$ (including SAT terms). Define the *entropy production density* $\sigma_{\text{diss}} := \frac{1}{T_{\text{eff}}}\, J^\top \Gamma J \geq 0$, where T_{eff} is an effective local temperature

(constant on stationary backgrounds, or provided by the thermodynamic closure used in a given application). Gate Drequires $\sigma_{\text{diss}} \geq 0$ a.e. and the ledger (6.13) to hold within numerical tolerances (see NG2).

Onsager reciprocity and colored channels.

When currents/forces are organized as $J = \mathsf{L}\,X$ with thermodynamic forces X, we enforce $\mathsf{L} = \mathsf{L}^\top \succeq 0$ (Onsager), so that $\Gamma = \mathsf{L}$ in (6.12). For *colored* channels obtained by a CP time–local embedding (pseudomodes), positivity of the auxiliary decay rates ensures $\Gamma \succeq 0$ at the macroscopic level, consistent with the GKSL layer (cf. V5, §??, §??).

Cone compatibility and causal damping

Dissipation must not "hide" superluminal transport. We therefore require that the principal (hyperbolic) part remains unchanged by Rayleigh terms and that the damping matrices Γ commute with the cone projectors of A_n (or, numerically, that the measured front speeds satisfy (6.8) with the same cEff). Equivalently, for each face normal n the block LMI

$$\mathcal{Q}_n(\lambda) \;:=\; \begin{bmatrix} \lambda H & \frac{1}{2}\,H A_n \\ \frac{1}{2}\,A_n^\top H & \Gamma \end{bmatrix} \;\succeq\; 0 \quad \text{for some } \lambda > 0, \tag{6.14}$$

is feasible on the DoV; this guarantees a *dissipative* numerical flux consistent with the cone (Gate G) and nonnegative damping (Gate D).

Discrete enforcement (NGate linkage)

For a one–step IMEX (or semi-implicit) scheme on time step Δt,

$$\mathcal{E}^{n+1} - \mathcal{E}^n \;\leq\; \Delta t\left(P^n_{\text{in}} - P^n_{\text{out}}\right) \;-\; \Delta t\left(J^n\right)^\top \Gamma J^n \;+\; \mathcal{O}(\Delta t^{p+1}), \tag{6.15}$$

with method order p. NG2 acceptance demands: (i) $\max_n \min(0, \sigma^n_{\text{diss}}) \geq -\varepsilon_D$, (ii) energy residual $\left|\Delta\mathcal{E} - \int (W_{\text{in}} - W_{\text{out}})dt\right| \leq \varepsilon_E$, and (iii) the discrete $J^\top \Gamma J$ scales with the observed order under refinement (NG3). The per–step ledger (6.15) is stored under `/audits/gate_D`.

Examples (templates)

- **Ohmic YM channel:** $J^A_i = E^A_i$, $\Gamma^{Ai,Bj} = \sigma^{AB}\delta_i{}^j$, $\sigma \succeq 0$; then $\mathcal{D} = \int E^A_i \sigma^{AB} E^B_i \, dV_{\text{eff}} \geq 0$.
- **Scalar diffusion:** $J = \nabla\Phi$, $\Gamma = \mathsf{D} \succeq 0$; $\mathcal{D} = \int (\nabla\Phi)^\top \mathsf{D}\,\nabla\Phi \, dV_{\text{eff}}$.
- **Frictional relaxation:** $J = U$, $\Gamma = \mathsf{M} \succeq 0$; yields exponential damping of high–frequency modes if $\lambda_{\min}(\mathsf{M}) > 0$.

HDF5 hooks for D

Under `/action/terms/rayleigh` store the tensors parameterizing Γ (e.g. `sigma`, `D`, `M`) with `units/axes`. Under `/audits/gate_D` store:

- `ledger`: arrays for $\mathcal{E}^n$, $P^n_{\text{in/out}}$, $\int J^\top \Gamma J$;
- `diss_nonneg`: minima of σ_{diss} over the run;
- `lmi_cone`: feasibility flags/margins for (6.14).

Acceptance 6.2. Gate D (Nonnegative dissipation) Pass iff *all* hold on the declared DoV:

1. **Positivity:** $\Gamma \succeq 0$ pointwise (or in the chosen basis), and $\sigma_{\text{diss}} \geq 0$ a.e.
2. **Ledger:** the discrete balance (6.15) holds within $(\varepsilon_D, \varepsilon_E)$ tolerances (NG2).
3. **Cone–compatibility:** LMI (6.14) feasible on all boundary patches, and measured front speeds satisfy (6.8).
4. **Replayability:** HDF5 entries reproduce the dissipation budget and acceptance margins on independent replay.

6.3 Unit system and scalings (M_*, L_*, T_*)

Objectives

We prescribe a deterministic map from physical variables to *dimensionless* unknowns so that (i) the PDE takes a symmetrizable–hyperbolic form with *unit* cone speed, (ii) all coefficients are $O(1)$ on the DoV, and (iii) CFL and convergence tests are portable across grids and backends. The map is recorded in HDF5 and is invertible.

Base unit system and reference scales

Let the global unit flag be `units_system` $\in$ {`SI`, `natural(c=hbar=1)`}. We fix *reference scales*

$$M_* \quad \text{(mass)}, \qquad L_* \quad \text{(length)}, \qquad T_* \quad \text{(time)}$$

and (optionally) a charge scale Q_* if the chosen unit system requires it. We also define the *effective light-speed scale*

$$\text{cEff}^* := \sup_{x \in \text{DoV}} \text{cEff}(x), \qquad \text{and set} \quad T_* = \frac{L_*}{\text{cEff}^*}, \tag{6.16}$$

so that the fastest front on the DoV travels with unit speed in the scaled variables.

Dimensional exponents and nondimensionalization

For any quantity Q with physical dimension $[Q] = M^a L^b T^c$ (and, if present, Q-charge exponent d), define its scaling factor

$$\sigma(Q) \;:=\; M_*^{\,a}\, L_*^{\,b}\, T_*^{\,c}\, Q_*^{\,d}, \qquad \widehat{Q} \;:=\; \frac{Q}{\sigma(Q)}.$$

Coordinates and derivatives:

$$x^i = L_*\, \widehat{x}^{\,i}, \qquad t = T_*\, \widehat{t}, \qquad \partial_i = \frac{1}{L_*}\, \widehat{\partial}_i, \qquad \partial_t = \frac{1}{T_*}\, \widehat{\partial}_{\widehat{t}}. \tag{6.17}$$

We treat g_{eff} as dimensionless (unit-bearing effective cones are carried by cEff). The volume element scales as $dV_{\text{eff}} = \sqrt{|\det g_{\text{eff}}|}\, d^3x = L_*^3\, d\widehat{V}$.

Field amplitudes (practical normalization).

Beyond dimensional exponents, we introduce *amplitude* scales to make kinetic blocks $O(1)$:

$$\Phi_*,\ \rho_*,\ \eta_*,\ T_*^{(\text{fld})},\ \mathcal{A}_* \quad \Rightarrow \quad \widehat{\Phi} = \Phi/\Phi_*,\ \ldots,\ \widehat{\mathcal{A}}_\mu = \mathcal{A}_\mu/\mathcal{A}_*.$$

For YM we adopt a coupling–absorbing normalization

$$\widehat{\mathcal{A}}_\mu \;:=\; g_{\text{ref}}\, \mathcal{A}_\mu, \qquad \widehat{\mathcal{F}}_{\mu\nu} \;:=\; g_{\text{ref}}\, L_*\, \mathcal{F}_{\mu\nu}, \tag{6.18}$$

with g_{ref} chosen at μ_{ref} (e.g. $g_2(\mu_{\text{ref}})$) so that the dimensionless Gram matrix in the YM block is well–conditioned. The spectral prefactor κ_G is recast into a dimensionless $\widehat{\kappa}_G$ accordingly.

Scaled PDE and energy

With U the state vector and H the symmetrizer (both re-expressed in hats), the evolution system (6.1) becomes

$$\widehat{\partial}_{\widehat{t}}\widehat{U} \;+\; \sum_{i=1}^{3} \widehat{A}^i(\widehat{U})\, \widehat{\partial}_i \widehat{U} \;=\; \widehat{S}(\widehat{U}) \;+\; \widehat{R}(\widehat{U}), \qquad \max_\alpha |\widehat{\lambda}_\alpha| \;\le\; 1, \tag{6.19}$$

where $\widehat{A}^i := T_* A^i / L_*$, so the cone bound is normalized to unity by the choice (6.16). The symmetrized energy becomes

$$\widehat{\mathcal{E}}[\widehat{U}] \;=\; \tfrac{1}{2} \int \widehat{U}^\top \widehat{H}(\widehat{U})\, \widehat{U}\; d\widehat{V}, \qquad \widehat{H} := H \text{ (re-expressed in hats).}$$

Dissipation adopts the quadratic form $\widehat{\mathcal{D}} = \int \widehat{J}^\top \widehat{\Gamma}\, \widehat{J}\; d\widehat{V}$ with $\widehat{\Gamma} := T_*\, \Gamma$ or the appropriate similarity transform depending on the definition of J.

CFL and convergence in hat variables

Let $\widehat{h}$ be the mesh scale in $\widehat{x}$ and $\widehat{\lambda}_{\max}$ the maximal characteristic speed (now ≤ 1 by construction). Then the CFL condition is

$$\Delta\widehat{t} \;\leq\; \mathrm{CFL}\,\frac{\widehat{h}}{\widehat{\lambda}_{\max}} \;\leq\; \mathrm{CFL}\,\widehat{h}. \tag{6.20}$$

Convergence and GCI (NG3) are computed directly on $\widehat{U}$ and the QoIs cast in hats; physical units are recovered a posteriori by multiplying with the appropriate $\sigma(Q)$.

Choice of (M_*, L_*) and balancing

A robust default is:

L_* set by the smallest physical length on the DoV (e.g. soliton core or FRW sound horizon), M_* s

Then tune $(\Phi_*, \rho_*, \eta_*, \mathcal{A}_*)$ so that the principal–part coefficients in (6.19) cluster near 1 and the Rayleigh entries $\widehat{\Gamma}$ fall in $[10^{-2}, 10^{+2}]$ on the DoV.

HDF5 schema hooks

Under the root attributes: `units_system`, `build_time_utc`. Store the scalings under `/scales`:

- `M_star`, `L_star`, `T_star`, (`Q_star` if used);
- `c_eff_star` = cEff* used in (6.16);
- `field_amps`: $\{\Phi_*, \rho_*, \eta_*, T_*^{(\mathrm{fld})}, \mathcal{A}_*\}$;
- `g_ref` and the dictionary of dimensionless numbers created by the scaling (e.g. $\widehat{\kappa}_G$, entries of $\widehat{\Gamma}$ ranges).

All datasets carry `units` (strings) and `axes` when array–valued.

Audit and replay

Every numerical artifact (background tables, linear spectra, solitons) must state its `/scales` snapshot. On replay, the pipeline (i) reconstructs (6.19) from hats, (ii) verifies the cone bound $\max|\widehat{\lambda}_\alpha| \leq 1$, and (iii) reconverts to physical units for reporting.

Acceptance.

Acceptance 6.3. Scaled IBVP acceptance A computation passes iff:

1. **Symmetrizable hyperbolic form:** the hatted system (6.19) admits $\widehat{H} \succeq \widehat{h}_0 \mathbb{I}$ with $(\widehat{H}\widehat{A}^i)$ symmetric;
2. **CFL bounds:** the run obeys (6.20) with recorded $\widehat{\lambda}_{\max} \leq 1$ and documented $\mathrm{CFL} \in (0, 1)$ (NG1);
3. **Convergence tests:** observed orders and GCIs in hat variables satisfy NG3 on all QoIs, and physical reconstructions match within reported tolerances.

Chapter 7

Numerics Gate (NGate): Well-Posedness, Accuracy, and Reproducibility

7.1 Purpose

The Numerics Gate (NGate) certifies that all computed results (backgrounds, linear modes, solitons, FRW runs) are *numerically trustworthy* within the declared DoV. It couples analytic well-posedness to discrete stability, convergence, causality, and reproducibility, and emits machine-checkable artifacts.

7.2 Discretization Classes and Recommended Schemes

- **Hyperbolic blocks** (wave-like terms): SBP–SAT finite differences or discontinuous Galerkin (DG) with entropy-stable Riemann fluxes.
- **Dissipative blocks** (Rayleigh/viscous terms): IMEX Runge–Kutta (e.g. ARK(4,4,3)) or Crank–Nicolson with stabilization; time step set by CFL plus a stiffness estimate.
- **Elliptic constraints** (projectors, Poisson-like): multigrid-preconditioned Krylov (CG/GMRES) with residual and compatibility checks.

7.3 Metrics and Acceptance Criteria

We define eight mandatory checks NG1–NG8. A run passes NGate iff all pass.

Checklist 7.1: NGATE

- **NG1 (CFL/Hyperbolicity).** Use $\Delta t \le \mathrm{CFL}\,\min_{\mathrm{cells}} \frac{h}{\lambda_{\max}}$, with $\lambda_{\max}$ the maximal characteristic speed from the discrete symbol. Record $\mathrm{CFL} \in (0,1)$ and grid h. *Accept if* the CFL constraint holds at all steps and no eigenvalue leaves the real axis (symmetrizable form).
- **NG2 (Discrete Energy/Dissipation).** There exists a discrete energy E^n with $E^{n+1} - E^n \le \Delta t\,(W^n_{\mathrm{in}} - W^n_{\mathrm{out}})$ and a nonnegative discrete entropy production $\sigma^n_{\mathrm{diss}} \ge 0$. *Accept if* $\max_n \min(0, \sigma^n_{\mathrm{diss}}) \ge -\varepsilon_D$ and $\left|\Delta E - \int (W_{\mathrm{in}} - W_{\mathrm{out}})\,dt\right| \le \varepsilon_E$.
- **NG3 (Order-of-Accuracy & GCI).** Perform refinements $h_1 > h_2 (> h_3)$ and estimate p_{obs} by Richardson. Grid Convergence Index:

$$\mathrm{GCI}_{12} = F_s \frac{|q_1 - q_2|}{|q_1|} \frac{1}{r^p - 1}, \qquad r = \frac{h_1}{h_2},\ F_s = 1.25.$$

  Accept if $0.8\,p_{\mathrm{method}} \le p_{\mathrm{obs}} \le 1.2\,p_{\mathrm{method}}$ and $\mathrm{GCI}_{12} \le \varepsilon_{\mathrm{GCI}}$ on all QoIs.
- **NG4 (Manufactured Solutions or Analytic Baselines).** Validate at least one block per class using MMS (injecting a source S so that an exact $\tilde{u}$ is recovered). *Accept if* errors scale as $O(h^{p_{\mathrm{method}}})$ in L^2 and L^∞.
- **NG5 (Constraint Preservation).** Gauge/BRST/Bianchi or Gauss-law constraints $C[u] = 0$ satisfy $\|C[u^n]\|_2 \le \varepsilon_{\mathrm{cons}}$ and converge at the primary order.
- **NG6 (Causality Fronts).** Front speeds from Riemann/packet tests satisfy $\max(v_{\mathrm{front}}/c_{\mathrm{eff}}) \le 1 + \varepsilon_G$ on DoVslices (see `/cones/lmi`).
- **NG7 (Solver Tolerances & Residuals).** Newton–Krylov reductions $\ge 10^8$ or below $\mathrm{tol}_{\mathrm{abs}}$; linear solves meet $\|\mathbf{r}\|/\|\mathbf{b}\| \le \mathrm{tol}_{\mathrm{lin}}$.
- **NG8 (Reproducibility & Determinism).** Fix RNG seeds and record `/runs` metadata (solver, grid, `cfl`, `tol_abs`, `tol_rel`, `seed`). Independent replays must match key metrics within $\varepsilon_{\mathrm{replay}}$.

Acceptance 7.1. Numerics Gate Acceptance A computation passes **NGate** iff NG1–NG8 all pass and the audit pack is written to `/audits` with `gate_N`=1 (see §7.7).

7.4 Default Thresholds

Parameter	Default	Comment
ε_D	10^{-12}	Entropy negativity margin (per step).
ε_E	10^{-6}	Discrete energy balance tolerance (relative).
$\varepsilon_{\mathrm{GCI}}$	2%	Max GCI on QoIs.
$\varepsilon_{\mathrm{cons}}$	10^{-8}	Constraint L^2 ceiling (nondimensional).
ε_G	10^{-3}	Front-speed margin vs. c_{eff}.
$\mathrm{tol}_{\mathrm{abs}}$	10^{-9}	Nonlinear residual absolute tol.
$\mathrm{tol}_{\mathrm{rel}}$	10^{-7}	Nonlinear residual relative tol.
$\mathrm{tol}_{\mathrm{lin}}$	10^{-10}	Linear solve relative residual.
$\varepsilon_{\mathrm{replay}}$	10^{-6}	Replay agreement on summary metrics.

7.5 Boundary Conditions and Filters

SAT penalties enforce characteristic inflow/outflow; entropy-stable fluxes are used at interfaces. For spectral/collocation methods, apply a *conservative* high-wavenumber filter that preserves discrete energy bounds (document cutoff and strength in `/runs/grid`).

7.6 Minimal Test Suite

- Riemann/packet fronts (NG6) and MMS per block (NG4).
- FRW spin-up energy/entropy balance (NG2) with $H(a)$ tables.
- Soliton relaxation (energy monotonicity, topology certificate).
- Linear mode table (phase/group velocities vs. analytic symbol).

7.7 HDF5 Integration

Store numerics audits under `/audits`:

- `gate_N` (uint8): 1/0 pass flag.
- `ngate/log` (string): human-readable summary.
- `ngate/metrics`: datasets for GCI, energy residuals, constraint norms, front-speed ratios.
- `/runs`: solver/grid/CFL/tolerances/seeds (as defined by the HDF5 v7 schema).

KPI–S2: CFL & Convergence & YAML Manifests

CFL scheme.

Choose the time step so that $\Delta t \leq \mathrm{CFL}\ \min_{\mathrm{cells}} h/\lambda_{\max}$. Reject runs with loss of symmetrizable hyperbolicity or $v_{\mathrm{front}}/c_{\mathrm{eff}} > 1 + \varepsilon_G$.

Convergence (Richardson/GCI).

Refine $h_1 > h_2 (> h_3)$, estimate p_{obs}, and require $0.8\, p_{\mathrm{method}} \leq p_{\mathrm{obs}} \leq 1.2\, p_{\mathrm{method}}$ and $\mathrm{GCI} \leq \varepsilon_{\mathrm{GCI}}$.

YAML manifests.

Ship human- and machine-readable YAML manifests; also embed them as plain text under `/runs/solver` and `/runs/grid` in the HDF5 file:

```
# /runs/solver (YAML)
name: usft-dg
version: 0.7.2
time_integrator: IMEX-ARK-4-4-3
cfl: 0.4
tol_abs: 1e-9
tol_rel: 1e-7
tol_lin: 1e-10
newton_maxit: 20
preconditioner: multigrid-amg
seed: 1337

# /runs/grid (YAML)
topology: cartesian
nx: 256
ny: 256
nz: 1
dx: 2.0e-3
dt: auto
filters:
  - type: spectral-cutoff
    k_cut: 0.7*k_nyquist
    conservative: true
bc:
  type: sat-entropy-stable
  inflow: characteristic
  outflow: characteristic
```

Chapter 8

Operational Background Solvers

Purpose. We define the *Operational Background Solvers* used across Volume VII to generate reproducible slices of the Domain of Validity (DoV) on which the full USFT PDE is integrated, audited, and benchmarked. Three canonical classes are shipped: (i) Minkowski (flat), (ii) FRW (homogeneous/isotropic cosmology), and (iii) stationary black–hole (BH) backgrounds. Each class comes with (a) a minimally parametrized effective metric g_{eff} and background controls, (b) a cone/characteristic audit, and (c) an HDF5 "background sheet" with unit tags, axes, and thresholds.

8.1 Minkowski, FRW, stationary BH (DoV slices)

Minkowski slice (flat, constant coefficients)

Metric and cones.

$$g_{\text{eff}}\mu\nu \equiv \text{diag}(-1, 1, 1, 1), \qquad \text{cEff} \equiv 1, \qquad dV_{\text{eff}} = d^3x.$$

The principal symbol $\mathsf{A}(n)$ of the first–order system (§6.1) is constant; all characteristic speeds satisfy $\max_\alpha |\lambda_\alpha(n)| \leq 1$ (Gate G). This slice is used for: (i) manufactured–solution tests (NG4), (ii) Riemann/packet fronts (NG6), and (iii) dispersion checks for linear modes.

Default boundary/ICs.

Periodic boxes for spectral/DG tests; otherwise, characteristic SAT inflow/outflow (6.9) with $g = 0$. Initial data are chosen to satisfy constraints to machine epsilon ($C(U) \approx 0$ at $t = 0$), see (6.4).

HDF5 keys.

`/backgrounds/minkowski/{params,cones}`:

- `params`: box sizes, grid, seed; `units`={length}.

- `cones`: arrays of $\lambda_{\max}$, front-speed ratios, LMI margins $\mu_{\min}$ over directions n.

FRW slice (homogeneous/isotropic cosmology)

Metric and controls.

$$ds^2 = -N^2(a)\,dt^2 + a(t)^2\,\gamma_{ij}\,dx^i dx^j, \qquad H(t) := \frac{\dot{a}}{a}, \qquad \mathrm{cEff}(a) \text{ from the kinetic cone of } g_{\mathrm{eff}}.$$

We operate either in cosmic time t or conformal time τ (stored as a flag). Background controls ship as tables $\{a,\, H(a),\, N(a),\, \mathrm{cEff}(a)\}$ with unit tags.

Linearized blocks (schematic).

For a scalar perturbation $\delta\Phi$ and a transverse YM mode $\mathcal{A}^T$,

$$\partial_t^2\delta\Phi + 3H\,\partial_t\delta\Phi - \frac{c_s^2}{a^2}\,\Delta\delta\Phi + m_{\mathrm{eff}}^2(a)\,\delta\Phi \;=\; \cdots\,,$$

$$\partial_t^2\mathcal{A}_i^T + H\,\partial_t\mathcal{A}_i^T - \frac{\mathrm{cEff}^2(a)}{a^2}\,\Delta\mathcal{A}_i^T \;=\; \cdots\,,$$

with dots denoting lower–order mixings from $(\rho, \eta, T, \chi^\mu)$ consistent with the DoV (hyperbolic principal part preserved). These formulas fix the *symbol* used for cone and CFL maps.

HDF5 keys.

`/backgrounds/frw/{params,profiles,cones}`:

- `params`: $\{N(a), H(a), \mathrm{cEff}(a)\}$ sampled on `/coords/a`.
- `profiles`: any background fields vs. a (e.g. $w_{\mathrm{eff}}(a)$).
- `cones`: front speeds $v_{\mathrm{front}}(a,k)$, $\mu_{\min}(a,k)$, CFL maps.

Stationary BH slice (excision–ready)

Metric and foliation.

Use a horizon–regular stationary chart (e.g. ingoing Eddington–Finkelstein):

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}\,(dx^i + \beta^i dt)(dx^j + \beta^j dt),$$

with lapse $\alpha(r)$, shift $\beta^i(r)$, and spatial metric $\gamma_{ij}(r)$. The effective cone speed $\mathrm{cEff}(r)$ is computed from the principal kinetic form.

Boundaries.

Inner boundary: excision at $r = r_{\mathrm{ex}} < r_h$ (pure outflow). Outer boundary: absorbing/SAT characteristic conditions with constraint damping ($\gamma_C > 0$ in (6.4)).

HDF5 keys.

`/backgrounds/bh_stationary/{params,cones}`:

- `params`: $\{r_h, r_{\text{ex}}, \alpha(r), \beta^i(r)\}$ on `/coords/r`.
- `cones`: $v_{\text{front}}(r,\theta)$, $\mu_{\min}(r,\theta)$, mask of outflow regions.

Global acceptance (per slice)

Acceptance 8.1. Background slice acceptance A background sheet passes iff: (i) cone feasibility $\mu_{\min} \geq 0$ on the slice; (ii) $\max v_{\text{front}}/\text{cEff} \leq 1 + \varepsilon_G$; (iii) constraint–propagation damping rates $\gamma_C \geq \gamma_{C,\min}$ are documented; (iv) HDF5 entries carry units/axes and reproduce the cone/CFL maps on replay.

8.1.1 Parameter posters and thresholds

Concept.

A *parameter poster* is a compact, machine–readable summary of a slice: baseline values, scan ranges, and hard thresholds (gates and DoV triggers). Posters are stored as YAML strings under each background group and mirrored as datasets for audit.

Poster fields (YAML sketch).

```
baseline:
  minkowski:
    box: [Lx, Ly, Lz]        # length units
    grid: [Nx, Ny, Nz]
    c_eff: 1.0
  frw:
    a_min: 1.0e-3
    a_max: 1.0
    H_of_a: table:/backgrounds/frw/params/H
    N_of_a: table:/backgrounds/frw/params/N
    c_eff_of_a: table:/backgrounds/frw/params/c_eff
  bh_stationary:
    r_h: ...
    r_ex: ...
    alpha_of_r: table:/backgrounds/bh_stationary/params/alpha
thresholds:
  gate_G:
    vmax_over_ceff: 1.001       # 1 + epsilon_G
    mu_min: 0.0
```

```
gate_S:
  H_min: 1.0e-6              # min eigenvalue of symmetrizer
gate_D:
  sigma_diss_min: 0.0
dov_triggers:
  cp_loss: true
  gap_closure: true
  cone_violation: true
```

Threshold definitions (audited).

- **Cone bound (Gate G):** $\mathcal{T}_G := \max_{\text{grid}}(v_{\text{front}}/\text{cEff}) \leq 1 + \varepsilon_G$, and $\mu_{\min} \geq 0$.
- **Energy positivity (Gate S):** $\lambda_{\min}(H) \geq \varepsilon_S > 0$ on the slice.
- **Dissipation (Gate D):** minimal entropy production $\sigma_{\text{diss}}^{\min} \geq 0$ for enabled channels.
- **DoV triggers:** flags indicate whether CP–loss, gap–closure, or cone–violation end validity.

HDF5 hooks.

Under each background:

- `poster_yaml` (string): the YAML poster.
- `thresholds/{vfront_over_ceff_max, mu_min, H_min, sigma_diss_min}`: scalars with units/tags.

AR.

HDF5 background sheets; cone plots; thresholds.

Chapter 9

Soliton Finder and Scattering Benchmarks

9.1 Energy minimization with topological constraints

Static energy functional (first principles)

On a background slice (§8.1), static configurations extremize the *energy* derived from the spectral Lagrangian by a $(3+1)$ split. In physical units:

$$E[U] = \int_\Omega \Big[\tfrac{1}{2}\,(\nabla_i^{\mathrm{eff}}\Phi)^\top \mathsf{Z}_\Phi(\Phi,\rho,\eta,T)\,(\nabla_i^{\mathrm{eff}}\Phi) + V(\Phi,\rho,\eta,T) \\ + \tfrac{1}{2}\,h_{AB}\,B_i^A B^{B\,i} + \tfrac{1}{2}\,(\nabla_i^{\mathrm{eff}}\rho)^\top \mathsf{Z}_\rho\,(\nabla_i^{\mathrm{eff}}\rho) + \tfrac{1}{2}\,(\nabla_i^{\mathrm{eff}}\eta)^\top \mathsf{Z}_\eta\,(\nabla_i^{\mathrm{eff}}\eta) + \cdots \Big]\, dV_{\mathrm{eff}}, \tag{9.1}$$

where $B^{A\,i} := \frac{1}{2}\,\epsilon^{ijk}\,\mathcal{F}^A_{jk}$, and the ellipsis covers model-specific cross-gradient terms permitted by DoV symmetries. In hatted variables (§6.3) the solver uses $\widehat{E}[\widehat{U}] = E[U]/E_*$ with E_* a recorded scale.

Stationary conditions (unconstrained).

The Euler–Lagrange equations for static extrema read

$$\frac{\delta E}{\delta U}(x) = 0 \qquad \Big(U = (\Phi,\rho,\eta,T,\mathcal{A}_i^A)\Big).$$

Gauge fixing is enforced by the generalized Lorenz condition (§6.1) with a static auxiliary; BRST/constraint residuals are required to vanish within tolerances.

Topological sectors and constraints

Topological charges partition configuration space into homotopy classes fixed by boundary conditions at spatial infinity (or on the outer boundary of Ω):

- **Flux/vortex number (2D plane or translational symmetry):** $Q_{\text{vort}} = \frac{1}{2\pi}\int_\Omega F_{12}\, d^2x \in \mathbb{Z}$.
- **Monopole charge (3D adjoint Higgs):** let $\hat{\Phi} := \Phi/|\Phi|$ on $\partial\Omega \simeq S^2$, then $Q_{\text{mon}} = \frac{1}{4\pi}\int_{\partial\Omega}\epsilon^{abc}\,\hat{\Phi}^a\partial_i\hat{\Phi}^b\partial_j\hat{\Phi}^c\, dS^{ij} \in \mathbb{Z}$.
- **Chern–Simons number (3D YM):** $N_{\text{CS}} = \frac{1}{8\pi^2}\int_\Omega \operatorname{Tr}\Big(A\wedge dA + \frac{2}{3}\,A\wedge A\wedge A\Big) \in \mathbb{R}/\mathbb{Z}$.
- **Hopf invariant (maps $S^3 \to S^2$):** $Q_{\text{Hopf}} = \frac{1}{(4\pi)^2}\int_\Omega \mathcal{A}\wedge\mathcal{F}$ for an Abelian representative of the pulled-back connection.

We *fix* a sector by prescribing integer targets $\mathbf{q} = (q_1, \ldots, q_m)$ for a set of charges $\{Q_k[U]\}$ computed by gauge-invariant discrete forms (DEC/finite-volume).

Constrained minimization: augmented Lagrangian

We minimize $E[U]$ subject to $Q_k[U] = q_k$ and (gauge/BRST) $C[U] = 0$. Introduce Lagrange multipliers λ_k and penalty weights $\beta_k > 0$:

$$\mathcal{L}_{\text{aug}}[U,\lambda] = E[U] + \sum_{k=1}^{m}\lambda_k\Big(Q_k[U]-q_k\Big) + \tfrac{1}{2}\sum_{k=1}^{m}\beta_k\Big(Q_k[U]-q_k\Big)^2 + \tfrac{1}{2}\,\beta_C\,\|C[U]\|_2^2. \quad (9.2)$$

The stationarity conditions are

$$\frac{\delta\mathcal{L}_{\text{aug}}}{\delta U} = 0, \qquad \frac{\partial\mathcal{L}_{\text{aug}}}{\partial\lambda_k} = Q_k[U] - q_k = 0.$$

In practice we use a *projected gradient flow* or a *Newton–Krylov* solve on $\nabla\mathcal{L}_{\text{aug}}$, with outer updates of (λ,β) (strategy below).

Gradient flows and gauge projection

Covariant (hat) gradient flow in fictitious time τ:

$$\partial_\tau\widehat{U} = -\,\widehat{\mathsf{G}}^{-1}\Big(\frac{\delta\widehat{E}}{\delta\widehat{U}} + \sum_k\lambda_k\,\frac{\delta\widehat{Q}_k}{\delta\widehat{U}} + \sum_k\beta_k\Big(\widehat{Q}_k - \widehat{q}_k\Big)\frac{\delta\widehat{Q}_k}{\delta\widehat{U}} + \beta_C\,\widehat{\mathcal{J}}_C^{\top}\widehat{C}\Big), \quad (9.3)$$

where $\widehat{\mathsf{G}}$ is a block preconditioner (kinetic metric), $\widehat{\mathcal{J}_C}$ is the Jacobian of constraints. Every n_{cool} steps we *project* to the gauge slice by solving

$$-\Delta_{\text{eff}}\,\alpha^A + \gamma_G\,\alpha^A = \mathcal{G}^A(\widehat{U}), \qquad \widehat{U} \;\mapsto\; \exp(i\alpha)\cdot\widehat{U},$$

("gauge cooling"), reducing the gauge residual $\mathcal{G}^A$ and improving Hessian conditioning.

Newton–Krylov solve (stationary system)

Define $F(U,\lambda) := \nabla\mathcal{L}_{\text{aug}}$ and solve $F = 0$ with a Jacobian–free Newton–Krylov (JFNK) method; at each iteration apply a trust region and a *constraint-preserving* linear solve (right-preconditioning by the kinetic blocks). Update $\lambda_k \leftarrow \lambda_k + \beta_k(Q_k[U] - q_k)$ and, if needed, increase $\beta_k\uparrow$ until $|Q_k - q_k| \le \varepsilon_Q$.

Discretization notes (DEC/DG)

Topological integrals use discrete exterior calculus to ensure metric-free integer structure on structured/unstructured meshes; energies/gradients use SBP–SAT finite differences or DG with entropy-stable fluxes (§7). Outer boundary implements the required homotopy class via fixed vacuum data on $\partial\Omega$ (Dirichlet on $\hat{\Phi}$, tangential gauge conditions).

Acceptance and certificates

Acceptance 9.1. Soliton acceptance A configuration $\widehat{U}_\star$ in sector $\mathbf{q}$ *passes* iff:

1. **Energy monotonicity:** the discrete ledger records $\widehat{E}^{(n+1)} \le \widehat{E}^{(n)}$ up to $\mathcal{O}(\Delta\tau^{p+1})$, and final decrease $< \varepsilon_E$ over the last N_{tail} steps.
2. **Topology certificate:** $\left|Q_k[\widehat{U}_\star] - q_k\right| \le \varepsilon_Q$ and the integer–rounded value is invariant under one refinement.
3. **Gauge/BRST/constraints:** $\|\widehat{C}[\widehat{U}_\star]\|_2 \le \varepsilon_C$; residual gauge $\|\mathcal{G}\|_2 \le \varepsilon_G$.
4. **Hessian positivity:** the smallest nontrivial eigenvalue of the constrained Hessian $\lambda^{\perp}_{\min} \ge 0$ (zero modes corresponding only to translations/rotations); report the count of zero modes and project them in scattering runs.
5. **Cone compliance:** local linearization respects Gate G(front-speed ratios, LMI margins) on the neighborhood of $\widehat{U}_\star$.

HDF5 outputs (`/solitons`)

For each sector and run, store:

- `energy`: $\widehat{E}$, E (with units), and ledger time series.

- `charges`: $\{Q_k\}$ raw and rounded, tolerances ε_Q.
- `fields`: compressed snapshots of $\widehat{U}_\star$ (profiles/slices).
- `residuals`: $\|\nabla\widehat{E}\|$, $\|C\|$, $\|\mathcal{G}\|$ histories.
- `hessian`: few smallest eigenvalues, mode flags (zero/negatives).
- `poster`: YAML with sector $\mathbf{q}$, mesh, solver, preconditioners.

Algorithmic sketch (gradient–projection)

1. Initialize $U^{(0)}$ by analytic ansatz in sector $\mathbf{q}$; set (λ, β).
2. Repeat: compute $G := \nabla\mathcal{L}_{\text{aug}}[U^{(n)}, \lambda]$, take preconditioned step (line search/trust region), project by gauge cooling every n_{cool} steps, update $\lambda \leftarrow \lambda + \beta(Q - q)$.
3. Stop when $\|G\| \le \varepsilon_{\text{stat}}$, $\|C\| \le \varepsilon_C$, and energy decrease below ε_E; emit certificates.

9.2 Newton–Krylov/gradient-flow pipelines

PDE view (relaxation) vs. variational view (minimization)

We implement two complementary pipelines for constructing solitons and benchmarking their neighborhoods:

P1. **Gradient-flow (GF) minimization**, i.e. the covariant flow (9.3) on the hatted energy $\widehat{E}[\widehat{U}]$ with augmented constraints (charges and gauge/BRST).

P2. **JFNK solve of the stationary system**, i.e. $F(U, \lambda) = 0$ where $F := \nabla\mathcal{L}_{\text{aug}}$ from (9.2).

GF is robust and topology-preserving; JFNK is fast near the solution and certifies second–order stationarity (Hessian information).

GF pipeline (algorithmic details).

Let τ be the fictitious time and $\Delta\tau$ the step. With a block preconditioner $\widehat{\mathsf{G}} \approx \operatorname{diag}(\mathsf{K}_\Phi, \mathsf{K}_\rho, \ldots)$ (kinetic blocks):

1. Compute the augmented gradient $G = \delta\widehat{E}/\delta\widehat{U} + \sum_k \lambda_k\, \delta\widehat{Q}_k/\delta\widehat{U} + \sum_k \beta_k(\widehat{Q}_k - \widehat{q}_k)\, \delta\widehat{Q}_k/\delta\widehat{U} + \beta_C\, \widehat{\mathcal{J}}_C^\top \widehat{C}$.
2. Take an IMEX step: $\widehat{U}^{n+1} = \widehat{U}^n - \Delta\tau\, \widehat{\mathsf{G}}^{-1} G^n$, with implicit treatment of stiff linear parts (e.g. Laplacians) to enlarge $\Delta\tau$.
3. Every n_{cool} steps solve the gauge-cooling Poisson problem and project back to the gauge slice (reduces $\|\mathcal{G}\|_2$).

4. Update multipliers: $\lambda_k \leftarrow \lambda_k + \beta_k(\widehat{Q}_k - \widehat{q}_k)$. If $|\widehat{Q}_k - \widehat{q}_k| > \varepsilon_Q$ after n_{outer} cycles, increase $\beta_k \uparrow$.

5. Stop when $\|G\|_2 \leq \varepsilon_{\text{stat}}$, energy tail decrease $< \varepsilon_E$, and $\|C\|_2 \leq \varepsilon_C$ (cf. Acceptance 9.1).

Time adaptivity: choose $\Delta\tau$ by a local energy decrease model; backtrack if $\widehat{E}^{n+1} > \widehat{E}^n$ (Armijo rule).

JFNK pipeline (equations and linear algebra).

Solve $F(U, \lambda) = 0$ with a Newton iteration

$$\begin{bmatrix} \partial_U F & \partial_\lambda F \\ \partial_U Q & 0 \end{bmatrix} \begin{bmatrix} \delta U \\ \delta\lambda \end{bmatrix} = - \begin{bmatrix} F \\ Q - q \end{bmatrix},$$

where $\partial_U F \approx$ constrained Hessian. We employ:

- **Jacobian-free** Krylov products: $(\partial_U F)\, v \approx \frac{F(U+\epsilon v) - F(U)}{\epsilon}$ with $\epsilon = \epsilon_{\text{diff}} \|U\| / \|v\|$.
- **Right-preconditioning** by kinetic blocks (multigrid/AMG on Laplacian-like parts), and *constraint-preserving* projections in the Krylov subspace.
- **Globalization** via trust region/line search to guarantee decrease of $\|F\|_2$; reject steps that increase the augmented energy.

Stopping rules: $\|F\| \leq \varepsilon_{\text{stat}}$, linear residual $\leq \text{tol}_{\text{lin}}$, no stagnation, constraint residual $\leq \varepsilon_C$. Emit the smallest few eigenpairs of the constrained Hessian (MINRES with shift-and-invert) for stability diagnostics.

HDF5 hooks (`/solitons/pipelines`).

Store `gf.yaml` and `jfnk.yaml` manifests (steps, tolerances, preconditioners), and `logs` (energy tail, norms, multiplier histories). Record pass/fail flags mirroring Acceptance 9.1 and NGate checks.

9.3 Cross sections σ/m and stability maps

Scattering setup and observables

Consider two solitons with total mass m_{tot} prepared at separation $R_0 \gg$ core size, relative speed v_{rel}, and impact parameter b. Simulate the time evolution on a Minkowski (or gently curved) slice with absorbing boundaries and SAT outflow.

Momentum-transfer and viscosity cross sections.

From differential cross section $d\sigma/d\Omega$ (estimated by angular histograms of outgoing energy/momentum flux at radius $R \gg$ core), define

$$\sigma_T := \int (1-\cos\theta)\,\frac{d\sigma}{d\Omega}\,d\Omega, \qquad \sigma_V := \int \sin^2\theta\,\frac{d\sigma}{d\Omega}\,d\Omega. \tag{9.4}$$

We report *per-mass* quantities σ_T/m_{tot} and σ_V/m_{tot} at the quoted v_{rel}. A discrete estimator using N_θ bins with solid angle $\Delta\Omega_j$ is

$$\widehat{\sigma}_T \approx \sum_{j=1}^{N_\theta}(1-\cos\theta_j)\,\frac{\Phi_j}{\Phi_{\text{inc}}}\,R^2\,\Delta\Omega_j, \qquad \Phi_j = \text{outgoing momentum flux in bin } j.$$

Capture/merger vs. elastic channels.

Classify outcomes by late-time diagnostics: (i) *elastic* (two cores out), (ii) *fusion* (single bound remnant with binding energy $\Delta E > 0$), (iii) *excitation* (internal-mode radiation). Mark capture regions in the (b, v_{rel}) plane; define the *total* cross section as $\sigma_{\text{cap}} = \pi b_c^2$ where b_c is the largest b that leads to capture.

Stability maps (Hessian and time-domain)

For each accepted soliton $\widehat{U}_\star$ (Acceptance 9.1):

(a) **Hessian (BdG) analysis:** compute the spectrum of the constrained Hessian $\mathcal{H}^\perp$ around $\widehat{U}_\star$; map $\lambda^\perp_{\min}(\mathbf{p})$ over parameter posters (couplings, background) to locate *linear-instability* regions ($\lambda^\perp_{\min} < 0$).

(b) **Time-domain poke tests:** perturb by a normalized eigenvector or random packet and evolve; declare *nonlinear stability* if the L^2 distance to the orbit (modulo symmetries) stays $< \varepsilon_{\text{nl}}$ for $t \in [0, T_{\text{hold}}]$.

(c) **Cone/LMI overlay:** superimpose Gate Gmargins (front-speed ratios and $\mu_{\min}$) to exclude causality-violating patches from the stability domain.

The result is a *stability map* over (couplings, background) with labels {stable, metastable, unstable} and overlays of capture bands.

HDF5 outputs (`/scattering` and `/stability`)

- `/scattering/runs/<id>`: setup (masses, v_{rel}, b), angular histograms, $\widehat{\sigma}_{T,V}$, channel classification, replay manifest.
- `/stability/maps`: grids of $\lambda^\perp_{\min}$, poke-test outcomes, overlays of Gate G, plus posters with thresholds.

- `/predictions/self_interact`: summary tables for σ_T/m_{tot}, σ_V/m_{tot} vs. v_{rel}.

Acceptance for scattering/stability

A run is *valid* iff: (i) cone and CFL audits pass on the domain; (ii) energy/momentum budgets close to within ε_E; (iii) angular flux is converged under $(R, \Delta\Omega)$ refinements (NG3); (iv) outcome classification is unchanged under one refinement.

Checklist 9.1: Soliton acceptance

- Energy/charge convergence and topology certificates.
- Ghost-free Hessians; Gcones respected.

Chapter 10

Linear Modes and Dispersion Extraction

Goal. Linearize the USFT field equations on a background slice (§8.1) and extract dispersion relations, characteristic cones, and stability certificates. We work with the quadratic action and its Euler–Lagrange linear system, making "ghost/gradient/causality" conditions explicit and audit-ready.

10.1 Perturbation matrices $(\mathsf{K}, \mathsf{Z}, \mathsf{M}^2)$

Quadratic action and block structure

Let $U = (\Phi, \rho, \eta, T, \mathcal{A}_\mu, \dots)$ and fix a smooth background U_0 on a chosen slice. Small perturbations δU enter the quadratic Lagrangian density (suppressing hats for readability):

$$\mathcal{L}^{(2)} \;=\; \tfrac{1}{2}\,\delta\dot{U}^\top\,\mathsf{K}\,\delta\dot{U} \;+\; \delta\dot{U}^\top\,\mathsf{Z}^i\,\partial_i\delta U \;-\; \tfrac{1}{2}\,(\partial_i\delta U)^\top\,\mathsf{G}^{ij}\,\partial_j\delta U \;-\; \tfrac{1}{2}\,\delta U^\top\,\mathsf{M}^2\,\delta U, \tag{10.1}$$

with:

- **Kinetic** matrix $\mathsf{K} = \mathsf{K}^\top \succeq 0$ (ghost-freeness).
- **Mixed** blocks Z^i (antisymmetric parts can be removed by local field redefinitions up to boundary terms).
- **Gradient** metric $\mathsf{G}^{ij} = \mathsf{G}^{ji}$ (positive semidefinite on the physical subspace).
- **Mass/Hessian** $\mathsf{M}^2 = \mathsf{M}^{2\top}$ (tachyon diagnostics).

Gauge and constraints are enforced either by gauge fixing (with BRST bookkeeping) or by projecting to the physical quotient before reading off blocks.

Linear equations and symbol.

The Euler–Lagrange equations take the form

$$\mathsf{K}\,\delta\ddot{U} \;+\; \Big(\mathsf{Z}^{i} - \mathsf{Z}^{i\,\top}\Big)\,\partial_i\delta\dot{U} \;-\; \partial_i\Big(\mathsf{G}^{ij}\,\partial_j\delta U\Big) \;+\; \mathsf{M}^2\,\delta U \;=\; 0. \tag{10.2}$$

For a plane-wave ansatz $\delta U \propto e^{-i\omega t + ik\,\hat{n}\cdot x}$, the *symbol* is the polynomial matrix

$$\mathsf{P}(\omega,k,\hat{n}) \;=\; -\omega^2\mathsf{K} + i\omega\,\mathsf{A}(\hat{n}) + k^2\,\mathsf{G}(\hat{n}) + \mathsf{M}^2, \quad \mathsf{A}(\hat{n}) := \Big(\mathsf{Z}^{i} - \mathsf{Z}^{i\,\top}\Big)\,\hat{n}_i,\; \mathsf{G}(\hat{n}) := \mathsf{G}^{ij}\,\hat{n}_i\hat{n}_j. \tag{10.3}$$

Dispersion branches satisfy $\det \mathsf{P}(\omega,k,\hat{n}) = 0$.

High-frequency (principal) cone.

In the principal limit $k \to \infty$ with $v := \omega/k$ fixed, lower-order M^2 and skew A drop out, yielding the generalized eigenproblem

$$\mathsf{G}(\hat{n})\,u \;=\; v^2\,\mathsf{K}\,u, \qquad v^2 \in \operatorname{spec}\Big(\mathsf{K}^{-1}\mathsf{G}(\hat{n})\Big). \tag{10.4}$$

The *front speeds* in direction $\hat{n}$ are $v_{\mathrm{front},\alpha}(\hat{n}) = \sqrt{v_\alpha^2(\hat{n})}$; causality requires $v_{\mathrm{front},\alpha}(\hat{n}) \le \mathrm{cEff}(\hat{n})$ on the DoV.

Diagnostics (ghost/gradient/tachyon)

$$\text{Ghost-free:}\quad \lambda_{\min}(\mathsf{K}) \;\ge\; 0, \qquad \text{(strictly} > 0 \text{ on the physical quotient)}, \tag{10.5}$$

$$\text{Gradient-stable:}\quad \mathsf{G}(\hat{n}) \;\succeq\; 0 \;\; \forall\hat{n}, \tag{10.6}$$

$$\text{No tachyons (IR):}\quad \lambda_{\min}\Big(\mathsf{K}^{-1/2}\,\mathsf{M}^2\,\mathsf{K}^{-1/2}\Big) \;\ge\; 0. \tag{10.7}$$

Inequalities are enforced after projecting out pure-gauge directions.

HDF5 hooks (`/linear`)

Store per background/poster:

- `K`, `Z_i`, `G_ij`, `M2` (with `axes` tags).
- `cones/v_front`($\hat{n}$) (grids over angles) and `cones/mu_min` (defined below).
- `gaps/gamma_min`: minimal positive eigenvalue of the BdG/Hessian on the quotient.

10.1.1 Mode decomposition; characteristic cones

Canonical kinetic frame.

Choose S such that $\mathsf{S}^\top \mathsf{K}\,\mathsf{S} = \mathbb{I}$. In variables $\widetilde{U} = \mathsf{S}^{-1}\delta U$,

$$\widetilde{\mathsf{P}}(\omega, k, \hat{n}) = -\omega^2 \mathbb{I} + i\omega\,\widetilde{\mathsf{A}}(\hat{n}) + k^2\,\widetilde{\mathsf{G}}(\hat{n}) + \widetilde{\mathsf{M}}^2, \quad \widetilde{\mathsf{G}} = \mathsf{S}^\top \mathsf{G}\,\mathsf{S},\ \widetilde{\mathsf{A}} = \mathsf{S}^\top \mathsf{A}\,\mathsf{S}. \tag{10.8}$$

The principal cone reduces to the ordinary eigenproblem $\widetilde{\mathsf{G}}(\hat{n})\,u = v^2\,u$.

Helicity/polarization split.

When symmetries are available (e.g. isotropy on Minkowski/FRW), decompose $\widetilde{\mathsf{G}}(\hat{n})$ into scalar/longitudinal/transverse blocks; compute v_{front} per helicity and compare to cEff.

Group velocity and subprincipal corrections.

At finite k, solve $\det \widetilde{\mathsf{P}} = 0$ for $\omega_\alpha(k, \hat{n})$ and extract $v_{g,\alpha} = \partial\omega_\alpha/\partial k$. Skew blocks $\widetilde{\mathsf{A}}$ shift phases but do not affect the principal cone.

10.2 Stability maps and LMI certificates

Cone LMIs (Gate G)

Define the *cone LMI* in direction $\hat{n}$:

$$\mathsf{C}(\hat{n}) \;:=\; \mathrm{cEff}(\hat{n})^2\,\mathsf{K} \;-\; \mathsf{G}(\hat{n}). \tag{10.9}$$

Causality is certified if $\mathsf{C}(\hat{n}) \succeq 0$ for all $\hat{n}$. Record the *margin*

$$\mu_{\min}(\hat{n}) \;:=\; \lambda_{\min}\Big(\mathsf{C}(\hat{n})\Big), \qquad \mu_{\min} \;\geq\; 0 \;\;\text{(pass)}. \tag{10.10}$$

Ghost/gradient LMIs

Enforce

$$\mathsf{K} \succeq 0, \qquad \mathsf{G}(\hat{n}) \succeq 0 \;\;\forall \hat{n}, \qquad \mathsf{H}_{\mathrm{IR}} := \mathsf{K}^{-1/2}\,\mathsf{M}^2\,\mathsf{K}^{-1/2} \succeq 0, \tag{10.11}$$

on the physical quotient. Near boundaries of the DoV, allow documented slack ε and track violations as DoV triggers.

Dispersion extraction and maps

For each poster (parameter point) and a grid $\{\hat{n}, k\}$:

1. Compute $v_{\text{front},\alpha}(\hat{n})$ from (10.4).

2. Solve $\det \widetilde{\mathsf{P}}(\omega, k, \hat{n}) = 0$ for $\omega_\alpha(k, \hat{n})$ (polynomial or contour methods); record $v_{g,\alpha}$ and attenuation.
3. Build *stability maps*: regions where (i) ghost/gradient LMIs pass, (ii) $\mu_{\min} \geq 0$, (iii) no tachyonic IR modes (nonnegative H_{IR}), (iv) spectral gap $\gamma_{\min} \geq \gamma_{\min}$ around the background.

PO.

Ghost/gradient/causality regions; $v_{\mathrm{front}} \leq \mathrm{cEff}$.

Checklist 10.1: Stability acceptance

- $\mathsf{K} \succeq 0$, spectral gaps $\geq \gamma_{\min}$ on posters.
- Cone LMI margins $\mu_{\min} \geq 0$ across the angle grid.

HDF5 outputs (`/linear` and `/cones`)

- `/linear/blocks`: K, G^{ij}, Z^i, M^2.
- `/cones/vfront`: $v_{\mathrm{front},\alpha}(\theta, \varphi)$; `/cones/mu_min`: $\mu_{\min}(\theta, \varphi)$.
- `/dispersion/branches`: $\omega_\alpha(k, \theta, \varphi)$ and $v_{g,\alpha}$.
- `/stability/map`: pass/fail masks for LMIs and gaps, with thresholds.

All datasets carry `units` and `axes`; posters include tolerance values $(\varepsilon, \gamma_{\min})$ used in the certificates.

Part IV

Quantization, Open Systems, and Born Limit

Chapter 11

BRST Quantization and Hadamard GNS

Aim. We construct the covariant classical phase space of the USFT fields on a globally hyperbolic background (M, g_{eff}) from the quadratic Lagrangian $\mathcal{L}^{(2)}$, identify the pre-symplectic structure via the Peierls construction, and show that, on the physical (gauge-quotiented) sector, the Peierls bracket reproduces the canonical (anti)commutation relations (CCR/CAR). This section prepares the ground for the Hadamard/GNS quantization and the pointer-sector analysis in later chapters.

11.1 Pre-symplectic reduction; Peierls ⇒ CCR/CAR

Setup and Green-hyperbolicity

Let $E \to M$ be the real (or $\mathbb{Z}_2$-graded) field bundle, with compactly supported smooth sections $\Gamma_0(E)$. The Euler–Lagrange linearization at a smooth background U_0 yields a linear differential operator

$$P : \Gamma(E) \longrightarrow \Gamma(E), \qquad \text{normally hyperbolic on } (M, g_{\text{eff}}). \tag{11.1}$$

Hypothesis (Green-hyperbolicity). P admits unique retarded/advanced fundamental solutions $G_{\text{R}}, G_{\text{A}} : \Gamma_0(E) \to \Gamma(E)$ with causal supports, and the *causal propagator*

$$\Delta \;:=\; G_{\text{R}} - G_{\text{A}} : \Gamma_0(E) \to \Gamma(E) \quad \text{satisfies} \quad P \circ \Delta \;=\; \Delta \circ P \;=\; 0. \tag{11.2}$$

Kinematic space, currents, and the pre-symplectic form

Let $\text{Sol}_{\text{sc}} := \{\phi \in \Gamma(E) \;:\; P\phi = 0,\ \text{spacelike-compact}\}$ and let $\mathcal{O}_{\text{lin}} := \{\phi \mapsto \langle f, \phi \rangle \;:\; f \in \Gamma_0(E)\}$ denote linear observables. The second variation defines a conserved bilinear

current $j^\mu(\varphi_1, \varphi_2)$ such that $\nabla_\mu j^\mu(\varphi_1, \varphi_2) = 0$ when $P\varphi_i = 0$. For any Cauchy surface Σ with unit normal n_μ,

$$\omega_\Sigma(\varphi_1, \varphi_2) \;:=\; \int_\Sigma j^\mu(\varphi_1, \varphi_2)\, n_\mu \, \mathrm{d}\Sigma \tag{11.3}$$

is independent of Σ and defines a *pre-symplectic* form on $\mathrm{Sol}_{\mathrm{sc}}$. Equivalently, for linear observables $F_f(\phi) = \langle f, \phi\rangle$ and $F_g(\phi) = \langle g, \phi\rangle$,

$$\omega(F_f, F_g) \;=\; \int_M \langle f, \Delta g\rangle \, \mathrm{dvol}_{g_{\mathrm{eff}}} \;=:\; \sigma([f], [g]), \qquad [f] := f + \mathrm{Im}\, P^*. \tag{11.4}$$

Here P^* is the formal adjoint w.r.t. $\mathrm{dvol}_{g_{\mathrm{eff}}}$, and σ is well-defined on the quotient $\Gamma_0(E)/\,\mathrm{Im}\, P^*$.

Gauge, constraints, and BRST reduction

Suppose gauge symmetries are generated by a differential operator $R : \Gamma(F) \to \Gamma(E)$ (for some gauge bundle $F \to M$) inducing the *Noether identity* $P \circ R = 0$. Then the degeneracy directions of ω are precisely $\mathrm{Ran}\, R$ (and their adjoint partners). Define the *physical kinematic space* as the BRST cohomological quotient at ghost number zero:

$$\mathcal{V}_{\mathrm{phys}} \;:=\; \ker P \Big/ \Big(\mathrm{Ran}\, R \,+\, \ker \omega\Big), \qquad \sigma_{\mathrm{phys}} \;\text{ induced by } \sigma. \tag{11.5}$$

In the minimal BRST complex, the differential s acts by $s\,\phi = Rc$ on fields and $s\,c = -\frac{1}{2}[c, c]$ on ghosts; the degree-zero cohomology $H^0(s)$ identifies gauge-invariant on-shell observables. The pairing σ descends to a *symplectic* form on $\mathcal{V}_{\mathrm{phys}}$.

Peierls bracket and microcausal functionals

Let $\mathfrak{F}_{\mathrm{mc}}$ denote the class of microcausal functionals (whose functional derivatives have wavefront sets avoiding problematic conormal directions). The *Peierls bracket* of $F, G \in \mathfrak{F}_{\mathrm{mc}}$ is

$$\{F, G\}_{\mathrm{P}} \;:=\; \Big\langle F^{(1)},\, \Delta\, G^{(1)} \Big\rangle, \qquad F^{(1)} := \frac{\delta F}{\delta \phi} \in \Gamma_0(E^*), \tag{11.6}$$

which on linear functionals reduces to (11.4). The bracket is antisymmetric, satisfies the Jacobi identity, and is causal w.r.t. g_{eff} (support properties inherited from Δ).

Lemma 11.1 (Peierls = covariant presymplectic)**.** *Under the hypotheses above, the Peierls bracket coincides with the inverse of the covariant pre-symplectic form on $\mathcal{V}_{\mathrm{phys}}$:*

$$\{F_f, F_g\}_{\mathrm{P}} \;=\; \sigma([f], [g]) \quad \textit{for all} \quad f, g \in \Gamma_0(E),$$

and the identity descends to the physical quotient (11.5). Proof sketch. *Use the advanced/retarded solutions to compute the linear response $F_f[\phi_\epsilon] - F_f[\phi]$ under a perturba-*

tion generated by the source coupled to $G_{\mathrm{R/A}}g$, and subtract the two responses; current conservation identifies this with (11.3). □

CCR/CAR algebras on the physical sector

Let $\mathcal{W}(\mathcal{V}_{\mathrm{phys}}, \sigma_{\mathrm{phys}})$ be the *Weyl C^*-algebra* generated by $\mathsf{W}(v)$, $v \in \mathcal{V}_{\mathrm{phys}}$, with relations

$$\mathsf{W}(v)\,\mathsf{W}(w) \;=\; e^{-\frac{i}{2}\,\sigma_{\mathrm{phys}}(v,w)}\,\mathsf{W}(v+w), \qquad \mathsf{W}(v)^* = \mathsf{W}(-v). \tag{11.7}$$

For fermionic (Grassmann-odd) sectors one obtains the *CAR algebra* generated by $\psi(f)$ with

$$\{\psi(f), \psi(g)^*\} \;=\; \langle f,\, S\,g\rangle, \qquad \{\psi(f), \psi(g)\} = 0, \tag{11.8}$$

where S is the appropriate (positive) two-point form induced by Δ and the choice of state in the next chapter. In both cases, by Lemma 11.1, the algebraic commutators/anticommutators reproduce the Peierls bracket on $\mathcal{V}_{\mathrm{phys}}$.

Proof obligations and DoV triggers

PO (quantization kinematics).

(i) Show P is Green-hyperbolic on the declared DoV; (ii) verify that $\ker\omega = \operatorname{Ran} R$ (no extra degeneracies); (iii) construct the physical quotient (11.5) and certify σ_{phys} is nondegenerate.

Triggers (end-of-validity).

Loss of Green-hyperbolicity (no $G_{\mathrm{R/A}}$), failure of current conservation, or appearance of non-gauge degeneracies in ω terminate the DoV for this construction and must be flagged in `/audits`.

The Hadamard state selection and GNS representation, ensuring positivity and microlocal spectrum condition, are developed next (see §11.2).

11.2 Hadamard two-point; Cauchy-surface independence

Hadamard condition and parametrix (microlocal spectrum)

Let $\widehat{\Phi}$ denote the (bosonic) quantum field on the CCR algebra built from the physical symplectic space (11.5). A state ω is *quasi-free Hadamard* if its two-point distribution

$$W_2(x,y) \;:=\; \omega\Big(\widehat{\Phi}(x)\widehat{\Phi}(y)\Big)$$

has the *Hadamard wavefront set* and local singular structure:

$$\mathrm{WF}(W_2) = \Big\{(x,k;y,-k') \in \mathcal{N}_x^+ \times \mathcal{N}_y^- \ : \ (x,k) \sim (y,k'),\ k \triangleright 0\Big\}, \tag{11.9}$$

and in any geodesically convex neighborhood,

$$\boxed{W_2(x,y) \ = \ \frac{1}{8\pi^2}\Big(\frac{U(x,y)}{\sigma_+(x,y)} + V(x,y)\,\log\frac{\sigma_+(x,y)}{\lambda^2} + W(x,y)\Big)} \tag{11.10}$$

with Synge world-function $\sigma = \frac{1}{2}\,(\text{geodesic distance})^2$, σ_+ the Feynman prescription, $U = \Delta_{VVM}^{1/2}$ (Van Vleck–Morette), and V, W smooth bi-tensors obtained from transport/recursion equations tied to the linearized operator P in (11.1). For vector/tensor/gauge fields, U, V, W are computed as parallel-transported sections with the Ehresmann/Yang–Mills connection of §3.1.1, and then projected to the BRST physical sector.

The antisymmetric part of any Hadamard W_2 is fixed by classical causality:

$$W_2(x,y) - W_2(y,x) \ = \ i\,\Delta(x,y), \qquad P_x W_2(x,y) = P_y W_2(x,y) = 0, \tag{11.11}$$

so choosing a Hadamard state amounts to fixing the *symmetric* bi-kernel subject to positivity.

Quasi-free positivity and GNS

Given a Hadamard W_2 as in (11.10)–(11.11), define for $f \in \Gamma_0(E)$ the smeared field $\widehat{\Phi}(f)$ and the quadratic form

$$\langle f, f\rangle_{W_2} \ := \ \int_{M\times M} \overline{f(x)}\, W_2(x,y)\, f(y)\, \mathrm{dvol}_{g_{\mathrm{eff}}}(x)\mathrm{dvol}_{g_{\mathrm{eff}}}(y) \ \geq 0. \tag{11.12}$$

The quasi-free state ω is uniquely specified by W_2 and yields, by the GNS construction, a Hilbert space $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$ with cyclic vector Ω_ω and field operators satisfying the CCR/CAR on the physical quotient (Lemma 11.1).

Proposition 11.2 (Hadamard ⇒ GNS positivity)**.** *Let W_2 be a bi-distribution obeying* (11.9)–(11.11)*. Then ω defined by W_2 is a positive, locally covariant, quasi-free state on the CCR/CAR algebra of the physical sector. In particular, the normal-ordered Wick polynomials defined by subtracting the parametrix in* (11.10) *are well-defined, and time-ordered products satisfy microlocal spectral constraints.* Proof sketch. *Microlocal spectrum ⇒ well-posed products; positivity* (11.12) *and* (11.11) *grant a positive-definite one-particle structure; complete the standard GNS steps.* □

Gauge/BRST sectors.

Choose a covariant gauge-fixing that renders P normally hyperbolic on each field component; construct W_2 blockwise and descend to the ghost#0 BRST cohomology (physical sector). Positivity is imposed only on the induced physical two-point function; unphysical degrees cancel in Wick polynomials by the BRST Ward identities (see §3.2).

Cauchy-surface independence

Let $j^\mu(\varphi_1, \varphi_2)$ be the conserved current of (11.3). For any two Cauchy surfaces $\Sigma_{1,2}$ enclosing a compact region containing $\operatorname{supp} f \cup \operatorname{supp} g$,

$$\omega_\Sigma(\varphi_1, \varphi_2) = \int_\Sigma j^\mu n_\mu \, \mathrm{d}\Sigma \quad \text{is independent of } \Sigma. \tag{11.13}$$

We now promote this kinematic independence to quantum expectation values.

Proposition 11.3 (Cauchy-surface independence of expectations)**.** *Let ω be a Hadamard state and A a polynomial (BRST-gauge-invariant) local observable smeared with a test function supported in a globally hyperbolic diamond $D(K)$. Then the vacuum expectation value $\langle \Omega_\omega, \pi_\omega(A)\Omega_\omega \rangle$ computed by quantizing on any Cauchy surface intersecting $D(K)$ is independent of the surface.* Proof sketch. *Use (i) the causal support properties of Δ and of W_2 (Hadamard microlocal spectrum) to restrict contractions to $D(K)$; (ii) the current conservation underlying* (11.13) *to match the initial-data propagators across surfaces; (iii) the fact that renormalized Wick products depend only on the local parametrix, which is foliation-independent.* □

Interacting counterterms (remark).

Where needed (e.g. semiclassical backreaction), local counterterms are fixed by the Hadamard parametrix coefficients U, V; renormalization conditions may be stored alongside the run's HDF5 bundle (cf. §4).

PO (as flagged). Provide the formal statements/proofs of Lemma 11.1, Proposition 11.2, and Proposition 11.3 on the declared DoV, including: (i) construction of $G_{\mathrm{R/A}}$; (ii) Hadamard transport recursion for U, V; (iii) BRST descent and positivity on the physical sector.

AR (Automatic/algorithmic checks).

- *WF-set diagnostics:* verify microlocal spectrum by testing that W_2-Hadamard parametrix is C^∞ (estimate Sobolev norms on shrinking balls).
- *Parametrix residuals:* compute $P_x W_2$ and $P_y W_2$ numerically; record $\|PW_2\|$ vs. resolution in `/audits`.

- *Choi spectra (open channels):* for effective GKSL channels used in measurement/tomography, store Choi eigenvalues to certify CPTP margins (links to §12.2).

Triggers (end-of-validity).

Loss of Hadamard form (failure of the smooth remainder), violation of positivity (11.12), or foliation dependence detected in benchmark observables (e.g. equal-time two-point on different surfaces) terminate the DoV and must be flagged in `/audits/gate_TQ`.

Chapter 12

Local GKSL, Colored Embeddings, and No-Signalling

Aim. We pass from the covariant Hilbert/GNS structure to an *operational* description of preparation/measurement on (M, g_{eff}): local, completely–positive and trace–preserving (CPTP) channels, calibrated instruments/POVMs, and their consistency with the causal cone and TQ. Colored (non-Markov) embeddings and the no–signalling theorem follow in subsequent sections.

12.1 System–meter channels; instruments; POVMs

Microscopic dilation and locality

Let $\mathcal{H}_{\text{S}}$ be the system (a BRST–physical sector of fields restricted to a diamond $D(K) \subset M$) and $\mathcal{H}_{\text{M}}$ a meter/apparatus with pointer algebra $\mathcal{A}_M$. A microscopic, L_{loc}–local interaction is specified on a Cauchy slice Σ_t by

$$H_{\text{tot}}(t) \;=\; H_S(t) + H_M(t) + \int_{\Sigma_t} \mathrm{d}\Sigma \Big(\sum_a J_a(x,t) \otimes B_a(x,t) \Big), \qquad \operatorname{supp} J_a(\,\cdot\,,t) \subset \mathbb{B}_{L_{\text{loc}}}(x_a), \tag{12.1}$$

where J_a are system currents and B_a meter operators; supports are bounded inside spatial balls of radius L_{loc} w.r.t. g_{eff}. Under the usual Born–Markov–secular hypotheses on the meter's correlation times and a Hadamard/GNS preparation of the joint initial state, the reduced system evolution $\rho_t = \operatorname{Tr}_M U_{t,0}(\rho_0 \otimes \sigma_M) U_{t,0}^\dagger$ is *time–local GKSL*:

$$\dot{\rho}_t \;=\; \mathcal{L}_t(\rho_t) \;:=\; -i[H_{\text{eff}}(t), \rho_t] + \sum_\alpha \Big(L_\alpha(t) \rho_t L_\alpha^\dagger(t) - \tfrac{1}{2}\{L_\alpha^\dagger(t) L_\alpha(t), \rho_t\} \Big), \tag{12.2}$$

with $L_\alpha(t) = \int_{\Sigma_t} \ell_\alpha(x,t)\,\mathrm{d}\Sigma$ supported in L_{loc}–balls. The generator admits a *local density* $\mathcal{L}_t = \int_{\Sigma_t} \mathcal{L}_t(x)\,\mathrm{d}\Sigma$ so that

$\mathcal{L}_t(x)$ depends only on the germ of ρ_t on $\mathbb{B}_{L_{\mathrm{loc}}}(x)$ and obeys cone support w.r.t. g_{eff}.

CPTP structure: Kraus and Choi

For a finite time step Δt, the propagator $\Phi_{t\to t+\Delta t} = \mathrm{e}^{\int_t^{t+\Delta t} \mathcal{L}_s\,\mathrm{d}s}$ is CPTP and admits Kraus operators $\{K_\mu\}$:

$$\Phi(\rho) = \sum_\mu K_\mu\,\rho\,K_\mu^\dagger, \qquad \sum_\mu K_\mu^\dagger K_\mu = \mathbb{I}. \tag{12.3}$$

Equivalently, its *Choi operator* $J_\Phi = (\Phi\otimes\mathbb{I})(|\Omega\rangle\langle\Omega|) \succeq 0$ (with $|\Omega\rangle$ maximally entangled) and $\mathrm{Tr}_S\, J_\Phi = \mathbb{I}$ certify complete positivity and trace preservation. **Acceptance (TQ).** We store J_Φ eigenvalues and the CPTP margins in `/quantum/gksl` of the HDF5 bundle.

Instruments and POVMs (operational layer)

A (Heisenberg) *instrument* with classical outcome set $\mathscr{X}$ is a family of CP maps $\{\mathfrak{I}_X\}_{X\in\mathscr{X}}$ such that the total map $\Phi = \sum_X \mathfrak{I}_X$ is CPTP. In Schrödinger picture,

$$\mathfrak{I}_X(\rho) = \sum_{\mu\in\mathcal{I}(X)} K_{\mu X}\,\rho\,K_{\mu X}^\dagger, \qquad \sum_X \sum_{\mu\in\mathcal{I}(X)} K_{\mu X}^\dagger K_{\mu X} = \mathbb{I}. \tag{12.4}$$

The associated *POVM* is $E_X := \sum_{\mu\in\mathcal{I}(X)} K_{\mu X}^\dagger K_{\mu X}$ with $\sum_X E_X = \mathbb{I}$. For any pre–measurement state ρ,

$$p(X) = \mathrm{Tr}\left(\mathfrak{I}_X(\rho)\right) = \mathrm{Tr}(E_X\rho), \qquad \rho \mapsto \rho_{|X} = \frac{\mathfrak{I}_X(\rho)}{p(X)} \ \text{(a posteriori state)}. \tag{12.5}$$

Naimark dilation. Every POVM $\{E_X\}$ arises by a unitary U on $\mathcal{H}_{\mathrm{S}}\otimes\mathcal{H}_{\mathrm{M}}$ and a sharp pointer PVM $\{\Pi_X\}$ on $\mathcal{H}_{\mathrm{M}}$: $\mathfrak{I}_X(\rho) = \mathrm{Tr}_M\Big[(\mathbb{I}\otimes\Pi_X)\,U(\rho\otimes\sigma_M)U^\dagger\Big]$.

Pointer calibration on (M, g_{eff}).

Let $\{\rho_j^{\mathrm{cal}}\}$ be a tomographically complete set in the *pointer sector* (large (ρ,η) gap). Repeated runs yield frequencies $f_{X|j}$. The *response matrix* $\mathsf{R}_{Xj} := \mathrm{Tr}(E_X\rho_j^{\mathrm{cal}})$ is estimated by maximum likelihood with constraints $E_X \succeq 0$, $\sum_X E_X = \mathbb{I}$:

$$\widehat{E} := \arg\max_{\{E_X\}} \sum_{j,X} n_{X|j} \log \mathrm{Tr}(E_X\rho_j^{\mathrm{cal}}), \quad \text{s.t. } E_X \succeq 0,\ \sum_X E_X = \mathbb{I}.$$

Uncertainties ΔE_X are obtained from the Fisher information; both $\widehat{E}$ and ΔE are stored under `/quantum/gksl/povm`.

Locality and cone compliance

Let $\operatorname{supp} E_X \subset D(K)$ and $D(K_1), D(K_2)$ be spacelike separated diamonds. If the meter couplings in (12.1) have L_{loc} smaller than the spacelike separation margin, then the Heisenberg–evolved effects commute,

$$[E_X(t_1),\, F_Y(t_2)] = 0, \qquad \text{for } \; D(K_1) \text{ spacelike to } D(K_2), \tag{12.6}$$

and the joint outcome probabilities factorize (see §12.3). This condition is tested numerically by (i) verifying cone LMIs (§10.2) and (ii) bounding Lieb–Robinson–type speeds extracted from Φ.

Auditables and triggers

AR. (i) Choi spectra J_Φ and CPTP margins; (ii) calibrated POVMs $\widehat{E}_X$ with error budgets; (iii) locality radii and spacelike–commutator tests. **Triggers (end–of–validity).** Negative Choi eigenvalue beyond tolerance, POVM non–closure $\|\sum_X E_X - \mathbb{I}\| > \varepsilon$, or detected violations of (12.6) on declared spacelike pairs; all must be flagged under `/audits/gate_TQ` and `/audits/gate_G`.

12.2 Calibration of GKSL from simulations; Choi spectra

Tomographic data from the microscopic solver

Fix a DoV slice and a tomographically complete set of inputs $\{\rho_0^{(m)}\}_{m=1}^{d^2}$ on the system sector (pointer-stable window). Evolve each by the microscopic dynamics (system+meter, local coupling (12.1)) for a short step Δt to obtain $\rho_{\Delta t}^{(m)}$. Vectorize density matrices with $\operatorname{vec}(\cdot)$ in a fixed operator basis $\{F_j\}_{j=1}^{d^2}$ (orthonormal in Hilbert–Schmidt):

$$y^{(m)} \;=\; \frac{1}{\Delta t}\Big(\operatorname{vec}(\rho_{\Delta t}^{(m)}) - \operatorname{vec}(\rho_0^{(m)})\Big) \;\approx\; \mathbb{L}\operatorname{vec}(\rho_0^{(m)}) \;+\; \epsilon^{(m)},$$

which yields a linear regression $\mathbf{y} \approx \mathbb{L}\,\mathbf{X}$ for the (superoperator) generator $\mathbb{L}$.

GKSL parametrization and constrained fit

Write the time–local Lindbladian as

$$\mathcal{L}(\rho) = -i[H,\rho] \;+\; \sum_{j,k=1}^{N} C_{jk}\Big(F_j\,\rho\,F_k^\dagger - \tfrac{1}{2}\{F_k^\dagger F_j, \rho\}\Big), \qquad C \succeq 0, \tag{12.7}$$

with $\{F_j\}$ traceless and $\operatorname{Tr} F_j^\dagger F_k = \delta_{jk}$; the Hamiltonian $H = H^\dagger$ and the *Kossakowski matrix* C encode all dissipative channels. The trace–preserving constraint is linear:

$$\forall \rho : \quad \operatorname{Tr} \mathcal{L}(\rho) = 0 \iff \sum_{j,k} C_{jk} \, [F_k^\dagger, F_j] \; = \; 0. \tag{12.8}$$

We estimate (H, C) by the convex program

$$\begin{aligned} \min_{H=H^\dagger,\ C\succeq 0} \quad & \sum_m \left\| y^{(m)} - \mathbb{L}(H,C)\,\mathrm{vec}(\rho_0^{(m)}) \right\|_2^2 \\ \text{s.t.} \quad & (12.8) \text{ holds}, \quad \|C\|_{\mathrm{tr}} \le \Lambda, \quad \|H\|_2 \le \Lambda_H, \end{aligned} \tag{12.9}$$

with Λ, Λ_H regularizers. Solutions define (time–slice) Lindblad operators L_α via the spectral factorization $C = U \operatorname{diag}(\gamma_\alpha)\, U^\dagger$ and $L_\alpha = \sum_j U_{j\alpha} F_j$ with rates $\gamma_\alpha \ge 0$.

From generator to channel and Choi diagnostics

Exponentiate $\mathcal{L}$ over Δt (e.g. 4th–order Magnus/TCL) to obtain the CPTP map $\Phi_{\Delta t} = \exp(\Delta t\, \mathcal{L})$; form the Choi operator

$$J_\Phi = (\Phi_{\Delta t} \otimes \mathbb{I})\Big(|\Omega\rangle\langle\Omega|\Big) \succeq 0, \qquad \operatorname{Tr}_S J_\Phi = \mathbb{I}. \tag{12.10}$$

We record $\lambda_{\min}(J_\Phi)$, $\|\operatorname{Tr}_S J_\Phi - \mathbb{I}\|$, and a contractivity proxy

$$\kappa_1 \;:=\; \sup_{m,n} \frac{\|\Phi_{\Delta t}(\rho_0^{(m)} - \rho_0^{(n)})\|_1}{\|\rho_0^{(m)} - \rho_0^{(n)}\|_1} \;\le\; 1 + \varepsilon_{\mathrm{TQ}},$$

together with uncertainty envelopes from bootstrap resamples of the tomographic set. All spectra/margins are stored under `/quantum/gksl/{choi_eigs, choi_traces, kappa1}` and the fitted (H, C) under `/quantum/gksl/{H, C, L_ops, rates}` with units/tags.

Colored (non–Markov) embeddings.

If residuals indicate memory, we promote the fit to a *Markovian dilation* on $S+A$ (finite pseudomodes A) with GKSL generator $\mathcal{L}_{SA}$ such that for all Δt in the calibration window $\Phi_{\Delta t}^{\mathrm{eff}} = \operatorname{Tr}_A \circ \exp(\Delta t\, \mathcal{L}_{SA}) \circ \iota$ matches the microscopic channel within tolerance. Fitting variables now include ancilla couplings and $C_{SA} \succeq 0$, with the same CPTP/contractivity audits applied to $\Phi_{\Delta t}^{\mathrm{eff}}$.

Triggers (end of validity).

Any of: $\lambda_{\min}(J_\Phi) < -\varepsilon_{\mathrm{TQ}}$, $\kappa_1 > 1 + \varepsilon_{\mathrm{TQ}}$, or poor goodness-of-fit in (12.9) beyond χ^2 thresholds, flags a DoVbreach at `/audits/gate_TQ`.

12.3 No-signalling theorem on (M, g_{eff})

Statement (operational form)

Let $D_A, D_B \subset M$ be spacelike–separated diamonds (w.r.t. g_{eff}). Assume: (i) the reduced dynamics on each region is generated by a *local* GKSL of the form (12.2) with support bounded by L_{loc}; (ii) the joint initial state has no superluminal correlations beyond the causal cone (Hadamard/GNS and microcausality); (iii) instruments $\{\mathfrak{I}_x^A\}$ on D_A and $\{\mathfrak{I}_y^B\}$ on D_B are calibrated POVM instruments (§12.1). Then the marginal outcome probabilities at A do not depend on the choice at B:

$$\forall x,\ \forall\{\mathfrak{I}_y^B\}: \quad p_A(x\,|\,\mathfrak{I}^B) \;=\; \sum_y \mathrm{Tr}\Big(\mathfrak{I}_y^B \circ \mathfrak{I}_x^A(\rho)\Big) \;=\; \mathrm{Tr}\Big(\mathfrak{I}_x^A(\rho)\Big), \tag{12.11}$$

and likewise with $A \leftrightarrow B$. Hence, all observed correlators lie in the *no–signalling polytope* and obey device–independent (DI) bounds.

Proof sketch (local GKSL, cone support, commutation)

Let $\mathfrak{E}_x^A$ and $\mathfrak{E}_y^B$ denote Heisenberg–picture maps. Locality yields a density $\mathcal{L}(x)$ with support bounded by L_{loc} and, by hyperbolicity/cone LMIs, the evolved observable algebras satisfy

$$[\mathfrak{E}_x^A(\mathcal{O}_A),\ \mathfrak{E}_y^B(\mathcal{O}_B)] = 0 \quad \text{when} \quad D_A \perp D_B, \tag{12.12}$$

up to exponentially small Lieb–Robinson–type tails bounded by the cone margin. Using cyclicity of the trace and (12.12) one obtains (12.11). Colored embeddings preserve the argument because the GKSL dilation acts on $S+A$ with *local* couplings supported inside enlarged cones, and tracing out A cannot create spacelike influence.

Tsirelson saturation and DI checks

Under (12.11), two–party correlators obey DI constraints. For CHSH,

$$\beta_{\text{CHSH}} \;\le\; 2\sqrt{2} \quad \text{(Tsirelson)},$$

with possible colored–noise degradations captured by the calibrated channel. We verify (i) non–signalling linear equalities/inequalities on the empirical tables, (ii) β_{CHSH} (or SDP DI bounds) within the feasible region predicted by Φ (cf. V5 §**??**). Violations trigger a TQ/Gaudit and mark a DoV breach.

Acceptance (TQ).

A time–slice passes if all hold:

TQ1. CPTP margins. $\lambda_{\min}(J_\Phi) \geq -\varepsilon_{\mathrm{TQ}}$ and $\| \operatorname{Tr}_S J_\Phi - \mathbb{I}\| \leq \varepsilon_{\mathrm{TP}}$ on calibrated $\Phi_{\Delta t}$.

TQ2. Contractivity. $\kappa_1 \leq 1 + \varepsilon_{\mathrm{TQ}}$ on the tomographic grid.

TQ3. Locality. Cone LMIs satisfied and spacelike commutator tests $\|[\mathfrak{E}_x^A(\cdot), \mathfrak{E}_y^B(\cdot)]\| \leq \varepsilon_{\mathrm{G}}$ for separated diamonds.

TQ4. DI feasibility. Empirical correlators fall inside the predicted DI region; in particular, $\beta_{\mathrm{CHSH}} \leq 2\sqrt{2} + \varepsilon_{\mathrm{DI}}$.

All metrics are written to `/quantum/gksl` and audited under `/audits/gate_TQ`.

Chapter 13

Spectral Flooding ⇒ Born Limit (Operational)

13.1 Flooding kernels; energy-share ledgers; gaps

Setting.

Fix a calibrated pointer POVM $\{\Pi_i\}_{i=1}^m$ on a Hilbert space $\mathcal{H}$, and a local CPTP evolution ρ_t solving a (possibly time–dependent) GKSL–type equation

$$\dot{\rho}_t \;=\; -\,i\,[H_t,\rho_t] + \sum_\alpha \Big(L_{\alpha,t}\rho_t L_{\alpha,t}^\dagger - \tfrac{1}{2}\{L_{\alpha,t}^\dagger L_{\alpha,t},\rho_t\}\Big), \qquad \rho_0 \in \mathcal{D}(\mathcal{H}). \tag{13.1}$$

Decompose the state by pointer sectors: $\rho_t = \sum_{i,j}\Pi_i\rho_t\Pi_j$ and define the *energy/share ledger*

$$p_i(t) \;:=\; \mathrm{Tr}(\Pi_i\rho_t), \qquad \mathbf{p}(t) = (p_1(t),\dots,p_m(t))^\top, \qquad \sum_i p_i(t) = 1.$$

Balance equation and flooding kernels.

A direct computation from (13.1) yields a closed balance law for $\mathbf{p}$:

$$\dot{p}_i(t) \;=\; \sum_{j\neq i}\Big[K_{i\leftarrow j}(t)\,p_j(t) - K_{j\leftarrow i}(t)\,p_i(t)\Big], \qquad i=1,\dots,m, \tag{13.2}$$

with *flooding kernels* (off–diagonal rates) given by

$$K_{i\leftarrow j}(t) \;=\; \sum_\alpha \mathrm{Tr}\Big(\Pi_i L_{\alpha,t}\,\Pi_j L_{\alpha,t}^\dagger\Big) \;+\; R_{i\leftarrow j}(t). \tag{13.3}$$

The remainder $R_{i\leftarrow j}(t)$ collects Hamiltonian–induced transfers mediated by off–diagonals $\Pi_i\rho_t\Pi_j$; under calibrated dephasing $\|\Pi_i\rho_t\Pi_j\|_1 \le C\,e^{-\gamma_{\mathrm{deph}}t}$ $(i\neq j)$, $R_{i\leftarrow j}$ is exponentially

suppressed and the dynamics reduces to a classical Kolmogorov system with generator

$$\mathsf{K}_{ij}(t) = \begin{cases} K_{i\leftarrow j}(t) \geq 0, & i \neq j, \\ -\sum_{k\neq i} K_{k\leftarrow i}(t), & i = j, \end{cases} \qquad \sum_i \mathsf{K}_{ij}(t) = 0.$$

Colored (non-Markov) embeddings.

For a CP time–local embedding of a colored process, the rates are temporally filtered convolutions

$$K_{i\leftarrow j}(t) \;=\; \int_0^\infty \kappa_{i\leftarrow j}(s)\, w_t(s)\, \mathrm{d}s, \qquad \kappa_{i\leftarrow j}(s) = \sum_\alpha \mathrm{Tr}\Big(\Pi_i L_\alpha\, e^{\mathcal{L}s}[\Pi_j]\, L_\alpha^\dagger\Big), \tag{13.4}$$

where w_t is the (calibrated) filter determined by the pseudomode/ancilla embedding and $\mathcal{L}$ is the Lindbladian acting on observables in the Heisenberg picture. The positivity of $K_{i\leftarrow j}(t)$ follows from complete positivity of the embedding.

Spectral gap and mixing.

Assume the Kolmogorov operator $\mathsf{K}(t)$ is uniformly ergodic on the declared *domain of validity* (DoV): there exists $\gamma > 0$ and a unique stationary vector $\mathbf{p}_\infty \succ 0$ such that for the *frozen* generator $\overline{\mathsf{K}}$ (time average on a window longer than the color time) the spectral gap satisfies

$$\min\Big\{\Re\lambda : \lambda \in \mathrm{spec}(-\overline{\mathsf{K}}),\ \lambda \neq 0\Big\} \;\geq\; \gamma. \tag{13.5}$$

Then total–variation distance contracts at rate γ: $\|\mathbf{p}(t) - \mathbf{p}_\infty\|_{\mathrm{TV}} \leq C\, e^{-\gamma t}$. A Lyapunov functional is the classical relative entropy $D(\mathbf{p}\|\mathbf{p}_\infty) = \sum_i p_i \log(p_i/p_{i,\infty})$ with $\frac{\mathrm{d}}{\mathrm{d}t} D \leq -2\gamma\, D$.

Born identification (operational).

The stationary weights coincide with the pointer statistics of the stationary state of the quantum channel:

$$p_{i,\infty} \;=\; \mathrm{Tr}(\Pi_i\, \rho_\infty), \qquad \mathcal{L}^*[\rho_\infty] = 0. \tag{13.6}$$

Under (13.5), any preparation relaxes to the *Born limit* $\mathbf{p}(t) \to \mathbf{p}_\infty$ with mixing rate $\geq \gamma$.

Ledger view. Define the sector-energy (or weight) current $J_{i\leftrightarrow j}(t) = K_{i\leftarrow j}(t)p_j(t) - K_{j\leftarrow i}(t)p_i(t)$. Then (13.2) is a Kirchhoff law $\dot{p}_i = \sum_{j\neq i} J_{i\leftrightarrow j}$ and the total dissipation rate $\sigma(t) = \sum_{i<j} J_{i\leftrightarrow j} \log \frac{K_{i\leftarrow j}p_j}{K_{j\leftarrow i}p_i} \geq 0$ is nonnegative (classical H–theorem), consistent with Gate D.

13.2 Finite-time deviation bounds; calibration protocol

Empirical frequencies and martingales.

Given a measurement record of length N in the pointer POVM, $\widehat{p}_i := \frac{1}{N}\sum_{n=1}^{N} \mathbb{1}\{X_n = i\}$, we bound the deviation $\widehat{p}_i - p_{i,\infty}$ using the mixing gap γ (or correlation time $\tau_{\text{corr}} \sim \gamma^{-1}$).

Theorem 13.1 (Sub–Gaussian finite–time bound)**.** *Let the classical chain generated by* $\mathsf{K}(t)$ *be uniformly ergodic with gap* γ*. Then for every* $\varepsilon > 0$*,*

$$\mathbb{P}\Big(\big|\widehat{p}_i - p_{i,\infty}\big| > \varepsilon\Big) \;\le\; 2\exp\Big[-\frac{2\,\varepsilon^2 N_{\text{eff}}}{1}\Big], \qquad N_{\text{eff}} \;=\; \frac{N}{1 + 2\sum_{k\ge 1}\rho_i(k)} \;\gtrsim\; \frac{N}{1+2/\gamma}, \tag{13.7}$$

where $\rho_i(k)$ *are the lag–k autocorrelations of* $\mathbb{1}\{X_n = i\}$*.*

A sharper *Bernstein–Freedman* form uses the variance proxy $V_N = \sum_{n=1}^{N} \mathrm{Var}(\mathbb{1}\{X_n = i\} \mid \mathcal{F}_{n-1})$:

$$\mathbb{P}\Big(\big|\widehat{p}_i - p_{i,\infty}\big| > \varepsilon\Big) \;\le\; 2\exp\Big[-\frac{N_{\text{eff}}\,\varepsilon^2}{2(\sigma_i^2 + \frac{1}{3}\varepsilon)}\Big], \qquad \sigma_i^2 = p_{i,\infty}(1 - p_{i,\infty}). \tag{13.8}$$

Deviation budgets for time–continuous runs.

For a continuous record of duration T sampled above the correlation rate ($\Delta t \ll \tau_{\text{corr}}$), the effective count is $N_{\text{eff}} \approx T/\tau_{\text{corr}} \approx \gamma T$ and (13.7) becomes

$$\mathbb{P}\Big(\big|\widehat{p}_i(T) - p_{i,\infty}\big| > \varepsilon\Big) \;\lesssim\; 2\exp\Big[-2\,\gamma T\,\varepsilon^2\Big]. \tag{13.9}$$

Calibration protocol (operational).

1. **Channel/POVM calibration.** Using preparation routines, perform quantum process tomography (or GST) to fit a GKSL generator $\widehat{\mathcal{L}}$ and pointer projectors $\{\widehat{\Pi}_i\}$. Store Choi eigenvalues/margins and CPTP slacks under `/quantum/gksl`.

2. **Kernel extraction.** Compute $K_{i\leftarrow j}(t)$ from (13.3) (Markov) or (13.4) (colored), with filter functions recorded under `/action/terms/rayleigh` or `/quantum/gksl`.

3. **Gap audit.** Estimate the mixing gap $\widehat{\gamma}$ from the spectrum of $-\overline{\mathsf{K}}$ or from the slowest relaxation mode in $\mathbf{p}(t)$. Store $\widehat{\gamma}$ and confidence intervals in `/predictions` and `/audits/gate_D`.

4. **Born prediction.** Compute $\mathbf{p}_\infty$ via the stationary solution of $\overline{\mathsf{K}}^\top \mathbf{p}_\infty = 0$ and cross–check with $\mathrm{Tr}(\widehat{\Pi}_i \rho_\infty)$ from $\widehat{\mathcal{L}}^*$.

5. **Run & test.** Execute the sequence pack; form empirical $\widehat{\mathbf{p}}$ and apply (13.7)–(13.9) with N_{eff} (or γT). The run *passes* if all coordinates satisfy the chosen two–sided window $\left|\widehat{p}_i - p_{i,\infty}\right| \leq \varepsilon_i$ at confidence $1-\alpha$.
6. **Ledger write–back.** Write the full audit to HDF5: `/predictions/p_infty`, `/audits/gate_D` (gap, variance budgets, passed/failed flags), and the raw sequence hash.

Acceptance (Gate D/TQ).

(i) CPTP margins nonnegative (small negative slacks allowed within declared tolerance); (ii) a strictly positive mixing gap $\gamma \geq \gamma_{\min}$ on the DoV; and (iii) empirical frequencies satisfy the finite–time windows implied by (13.7) or (13.8) at the stated confidence. All objects (kernels, gaps, windows, manifests) are shipped in the public HDF5 bundle and reproduced by the CI replay.

Part V

Recovery and Matching of Known Physics

Chapter 14

GR & SM Limits

Aim. Show that, on a declared low–energy/low–curvature DoV, the Unified Spectral Field Theory (USFT) reduces to General Relativity plus the Standard Model (with threshold corrections) and that all extra operators are irrelevant in the EFT sense and quantitatively bounded by the DoVtolerances. We also spell out the operational matching conditions that connect the USFT parameters to the measured SM ones.

14.1 Low-energy decoupling: $\mathcal{L} \to \mathcal{L}_{\mathrm{GR}} + \mathcal{L}_{\mathrm{SM}}$

Regime and scalings.

Fix energy, length, and time scales (E, L, T) inside the DoVsuch that

$$E \ll \Lambda_{\mathrm{spec}}, \qquad |\mathrm{Riem}(g_{\mathrm{eff}})| \ll \Lambda_{\mathrm{curv}}^2, \qquad \partial/\Lambda_{\mathrm{spec}} \ll 1,$$

where Λ_{spec} is the smallest heavy spectral/mass threshold in the USFT spectrum and Λ_{curv} the curvature threshold extracted from the geometric block. All gates hold on this window: S (positivity/hyperbolicity), G (causal cone LMIs), TQ (CPTP), D (nonnegative entropy).

Heavy/light split and EFT map.

Split fields/descriptors $\mathcal{S}$ into light L (SM+graviton+light scalars) and heavy H (mass $\geq \Lambda_{\mathrm{spec}}$). The path–integral definition of the *low–energy effective Lagrangian* is

$$e^{i\int \mathcal{L}_{\mathrm{eff}}[L]} \;=\; \int \mathcal{D}H \;\exp\Big\{ i \int \mathcal{L}[L, H; g_{\mathrm{eff}}] \Big\}. \tag{14.1}$$

On the DoV, integrating out H yields a local derivative expansion organized by canonical mass dimension:

$$\mathcal{L}_{\mathrm{eff}} \;=\; \underbrace{\mathcal{L}_{\mathrm{GR}}[g_{\mathrm{eff}}] + \mathcal{L}_{\mathrm{SM}}[B_\mu, W_\mu^i, G_\mu^a, \psi, \varphi]}_{\text{renormalizable block}} \;+\; \sum_{d>4} \frac{1}{\Lambda^{\,d-4}} \sum_k c_{d,k}\, \mathcal{O}_k^{(d)}, \tag{14.2}$$

where the $\mathcal{O}_k^{(d)}$ are gauge– and diffeo–invariant operators built from the light sector; coefficients $c_{d,k}$ are finite thresholds determined by the heavy content and interactions.

GR block from the geometric sector.

The geometric part reduces to the Einstein–Hilbert form with positive Newton term and controlled higher–curvature corrections:

$$\mathcal{L}_{\rm GR} \;=\; \frac{M_P^2}{2}\,R(g_{\rm eff}) \;-\; \Lambda_{\rm cc} \;+\; \alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu} \;+\; \mathcal{O}(\partial^6/\Lambda_{\rm curv}^2), \tag{14.3}$$

with $M_P^2 > 0$ inferred from the quadratic expansion of the spectral action on the background (Gate S). On the DoV, the dimension–6 and higher terms are bounded by $\left|\alpha_{1,2}\right| |\mathrm{Riem}|/\Lambda_{\rm curv}^2 \ll 1$, hence numerically negligible.

SM block and canonical normalizations.

Using the YM–anchor relations derived earlier, the gauge–kinetic terms are canonically normalized:

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4}\left(\frac{1}{g_1^2}B_{\mu\nu}B^{\mu\nu} + \frac{1}{g_2^2}W^i_{\mu\nu}W^{i\,\mu\nu} + \frac{1}{g_3^2}G^a_{\mu\nu}G^{a\,\mu\nu}\right), \qquad g_i^{-2} = \kappa_G\,\Xi_i \;\;(\text{at } \mu_{\rm ref}), \tag{14.4}$$

with Ξ_i the Lie–metric projectors and κ_G the spectral normalization. Matter kinetic terms are brought to canonical form by positive field–space metrics (Gate S). Yukawa and scalar potentials inherit the USFT textures; at tree level the light/heavy split implies

$$\mathcal{L}_{\rm Yuk} + V(\varphi) \;=\; -\,\overline{\psi}_L\,Y_{\rm eff}\,\varphi\,\psi_R - V_{\rm SM}(\varphi) \;+\; \mathcal{O}(\Lambda^{-1}), \qquad Y_{\rm eff} = Y - X\,M_H^{-1}\,X^\dagger + \cdots\,, \tag{14.5}$$

which encodes seesaw/threshold corrections from heavy mediators with mass matrix M_H and mixings X.

Tree–level decoupling (algebraic heavy fields).

Whenever a heavy field appears without derivatives at leading order, its equation of motion is algebraic and gives the standard tree–level matching. For example, a heavy scalar H with quadratic potential $\mathcal{L} \supset -\frac{1}{2}H(\mathcal{M}^2 + \lambda\,\varphi^\dagger\varphi)H + H\,J(L)$ yields at leading order $H^\star = \mathcal{M}^{-2}J + \mathcal{O}(\Lambda^{-3})$ and contributes the dimension–6 operator $\frac{1}{2}J^\top\mathcal{M}^{-2}J$.

One–loop thresholds and RG continuity.

At scales $\mu < \Lambda_{\rm spec}$, couplings run with the SM β–functions. Matching at the decoupling scale $\mu = \mu_{\rm th}$ imposes

$$g_i^{-2}(\mu^-) \;=\; g_i^{-2}(\mu^+) + \Delta_i^{(\rm th)}, \qquad Y_{\rm eff}^{-1}(\mu^-) \;=\; Y^{-1}(\mu^+) + \Delta_Y^{(\rm th)}, \tag{14.6}$$

with finite Δ's computed from the heavy spectrum. The RG tables are stored under `/rg/flows` in the HDF5 bundle.

Decoupling proposition.

Proposition 14.1 (USFT→GR+SM on the low–energy DoV)**.** *Assume (i) Gate S(positive kinetic metrics, hyperbolicity) and Gate G(causal cones), (ii) a mass/curvature separation* $(E, \sqrt{|\mathrm{Riem}|}) \ll (\Lambda_{\mathrm{spec}}, \Lambda_{\mathrm{curv}})$*, and (iii) anomaly cancellation for the embedded gauge spectrum. Then the effective Lagrangian on the DoVis of the form* (14.2)*, with the renormalizable block equal to* $\mathcal{L}_{\mathrm{GR}} + \mathcal{L}_{\mathrm{SM}}$ *(up to field redefinitions), and the dimension–*$d > 4$ *operators suppressed by* $(E/\Lambda)^{d-4}$ *and* $(\sqrt{|\mathrm{Riem}|}/\Lambda_{\mathrm{curv}})^{d-4}$*, uniformly on the DoV.*

Operational acceptance (this section).

Acceptance 14.2. GR+SM decoupling

- **Positivity & cones (Gates S/G).** Einstein term $M_P^2 > 0$; matter/gravity kinetic blocks $\succ 0$; cone LMIs satisfied over the background sheets used in matching.
- **Threshold audit.** The finite sets $\{\Delta_i^{(\mathrm{th})}, \Delta_Y^{(\mathrm{th})}\}$ are computed and saved; the reported SM couplings at M_Z reproduce the experimental ellipsoids within stated uncertainties when evolved with `/rg/flows`.
- **Irrelevant operator ceiling.** For all used observables, the sum of $d > 4$ contributions is bounded by a user–declared tolerance (recorded under `/audits/gate_S`).

What is stored (HDF5).

`/backgrounds/{minkowski,frw}/params` (curvature bounds); `/rg/flows` (running and thresholds); `/predictions` (matched $g_{1,2,3}(M_Z)$, $\sin^2\theta_W(M_Z)$); `/audits/gate_S, gate_G` (positivity/LMIs) with pass flags.

14.2 Cosmological sector: modified FRW → ΛCDM

Background and bookkeeping.

Work on the spatially homogeneous/isotropic ansatz of the effective metric g_{eff},

$$\mathrm{d}s^2 \;=\; -\,\mathrm{d}t^2 + a(t)^2\,\gamma_{ij}\,\mathrm{d}x^i\mathrm{d}x^j, \qquad \gamma_{ij} \in \{\delta_{ij},\ \text{const. curvature } k\}, \tag{14.7}$$

with Hubble function $H \equiv \dot{a}/a$ and scale factor $a(t)$. Let the light matter sector be (ρ_m, p_m) and radiation $(\rho_r, p_r = \frac{1}{3}\rho_r)$, and denote by $(\rho_{\mathrm{X}}, p_{\mathrm{X}})$ the *USFT geometric/spectral* background contribution obtained from the conservative Lagrangian block by

$$T_{\mathrm{X}}^{\mu\nu} \;=\; \frac{2}{\sqrt{-g_{\mathrm{eff}}}}\,\frac{\delta}{\delta g_{\mu\nu}^{\mathrm{eff}}}\int \mathrm{d}^4x\,\sqrt{-g_{\mathrm{eff}}}\,\mathcal{L}_{\mathrm{cons}}[\mathcal{S}; g_{\mathrm{eff}}], \qquad \rho_{\mathrm{X}} = T^0{}_0, \quad p_{\mathrm{X}} = \tfrac{1}{3}T^i{}_i. \tag{14.8}$$

(Any Rayleigh/dissipative power terms are audited under Gate D and are set to zero for background evolution unless explicitly stated.)

Modified Friedmann system.

The background equations take the EFT form

$$3\,M_\star^2(a)\,H^2 = \rho_m + \rho_r + \rho_{\rm X}, \tag{14.9}$$

$$-\,2\,M_\star^2(a)\,\dot H = \Big(\rho_m + p_m\Big) + \Big(\rho_r + p_r\Big) + \Big(\rho_{\rm X} + p_{\rm X}\Big) \;-\; \dot M_\star^2(a)\,H, \tag{14.10}$$

where $M_\star^2(a)$ is the *effective Planck mass* extracted from the quadratic expansion of the geometric block around FRW (Gate S). The USFT background sector obeys the (possibly sourced) continuity law

$$\dot\rho_{\rm X} + 3H\Big(\rho_{\rm X} + p_{\rm X}\Big) \;=\; Q_{\rm X}, \qquad Q_{\rm X} = 0 \text{ in the conservative limit.} \tag{14.11}$$

Define the effective equation of state $w_{\rm eff}(a) := p_{\rm X}/\rho_{\rm X}$ whenever $\rho_{\rm X} > 0$.

ΛCDM limit and tolerances.

The reduction to ΛCDM is achieved when, on the declared cosmological DoV,

$$M_\star^2(a) = M_P^2 \quad \text{(constant)}, \qquad Q_{\rm X} = 0, \qquad p_{\rm X} = -\rho_{\rm X} \equiv -\Lambda_{\rm cc}\,M_P^2, \tag{14.12}$$

so that (14.9)–(14.10) become the standard Friedmann equations with cosmological constant $\Lambda_{\rm cc}$. We quantify deviations by bounded *EFT drift functions*

$$\alpha_M(a) := \frac{\mathrm{d}\ln M_\star^2}{\mathrm{d}\ln a}, \qquad \alpha_B(a),\ \alpha_K(a),\ \alpha_T(a)$$

(obtained from the linearized action around FRW, §10.1): the ΛCDM gate for background expansion requires $\big|\alpha_M\big|, \big|\alpha_B\big|, \big|\alpha_K\big|, \big|\alpha_T\big| \le \varepsilon_{\rm FRW}$ on the DoVgrid. In particular $\alpha_T = 0$ implies $c_T = 1$ at background level.

Operational $H(a)$ and derived baselines.

Given a choice of $\{\Omega_{m0}, \Omega_{r0}, \Omega_{k0}\}$ at $a = 1$ and the USFT background $(M_\star^2, \rho_{\rm X}, p_{\rm X})$, we integrate (14.9)–(14.11) to obtain $H(a)$ and

$$q(a) \equiv -\frac{\mathrm{d}}{\mathrm{d}\ln a}\Big(\frac{1}{H}\Big) - 1, \qquad w_{\rm eff}(a) = -1 - \frac{1}{3}\,\frac{\mathrm{d}\ln\rho_{\rm X}}{\mathrm{d}\ln a} \quad (\rho_{\rm X} > 0),$$

and tabulate the comoving distances and the sound horizon used by the SNe/BAO pipelines. These arrays are stored under `/backgrounds/frw/profiles` and `/predictions/weff` in the HDF5 bundle.

Linear scalar sector (growth and slip, summary).

At subhorizon scales the density contrast δ_m evolves as

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}(a,k)\,\rho_m\,\delta_m = 0, \qquad G_{\rm eff}(a,k) \;=\; \frac{1+\mu(a,k)}{M_\star^2(a)}\,\frac{1}{8\pi}, \tag{14.13}$$

with $\mu(a,k)$ and the lensing combination $\Sigma(a,k)$ determined by the EFT drift functions and the K,Z matrices in §10.1. The *slip* parameter $\eta_{\rm slip}(a,k) := \Phi/\Psi - 1$ is likewise $\mathcal{O}(\alpha)$ and vanishes in the ΛCDM gate.

Matching tables (EFT map, dimensional check).

For auditability we provide a compact mapping from USFT background functions to the EFT-of-DE set $\{M_\star^2, \Lambda, c, \alpha_K, \alpha_B, \alpha_M, \alpha_T\}$, together with units:

Symbol	Meaning / units
$M_\star^2(a)$	effective Planck mass from quadratic action (energy)^2
$\Lambda(a)$	background vacuum density in (14.9) (energy)^4
$c(a)$	braiding background term (energy)^4
$\alpha_{K,B,M,T}(a)$	dimensionless drifts (see §10.1)

Dimensional consistency of all entries is enforced in the `/audits` unit tests.

Acceptance (FRW→ ΛCDM).

Acceptance 14.3. Cosmology baselines

- **Dimensional/EFT checks (PO).** The matching table above is present in HDF5 with correct units; drift functions are computed from the linearized action and satisfy $\sup|\alpha_i| \le \varepsilon_{\rm FRW}$ on the DoVgrid.
- **Baselines reproduced.** With the declared priors $(\Omega_{m0}, \Omega_{b0}, h_0, \Omega_{k0})$, the expansion observables (SNe/BAO D_M, H, CMB θ_*) computed from $H(a)$ are within the published tolerance ellipsoids; the growth/lensing pair (μ, Σ) and slip $\eta_{\rm slip}$ are consistent with ΛCDM within the stated envelopes.
- **Storage.** Arrays $\{H(a), q(a), w_{\rm eff}(a)\}$, distances, and acceptance flags are written to `/backgrounds/frw` and `/audits/gate_S`.

Chapter 15

SM Mapping Dictionary

This chapter fixes the *matter dictionary* that maps the internal spectral fiber to the Standard Model (SM) representations and charges. Given the gauge embedding tensors ${E^A}_{(1)}, {E^A}_{(2)i}, {E^A}_{(3)a}$ and the Lie–metric h_{AB} (from §3 and the YM–anchor chapter), we specify: (i) the decomposition of the fiber into SM multiplets, (ii) the hypercharge normalization and the resulting electric charges, and (iii) machine–readable records (HDF5) for audits and anomaly checks (handled in §15.2).

15.1 Fiber → SM assignments & hypercharge normalization

Decomposition.

Let $\mathcal{F}$ denote the internal fiber carrying a (generally reducible) representation $\mathcal{R}$ of the gauge algebra $\mathfrak{g}$. Upon restricting $\mathcal{R}$ to the SM subalgebra $\mathfrak{g}_{\mathrm{SM}} = \mathfrak{u}(1)_Y \oplus \mathfrak{su}(2)_L \oplus \mathfrak{su}(3)_c$, the matter space decomposes into SM multiplets

$$\mathcal{F} \;=\; \bigoplus_{\alpha\in\mathsf{mult}} \Big(R_3^{(\alpha)},\, R_2^{(\alpha)}\Big)_{y_\alpha} \;\otimes\; \mathbb{C}^{\mathcal{N}_\alpha}, \qquad R_3^{(\alpha)} \subset SU(3)_c,\;\; R_2^{(\alpha)} \subset SU(2)_L, \tag{15.1}$$

where y_α is the hypercharge eigenvalue and $\mathcal{N}_\alpha$ the multiplicity (e.g. number of generations). Let $P^{(\alpha)} : \mathcal{F} \to \mathcal{F}$ be orthogonal projectors onto each multiplet; in practice we store sparse matrices for $\{P^{(\alpha)}\}$.

Generators on matter.

Denote by $T^a_{(3)}$, $T^i_{(2)}$, and Y the representation matrices of the color, weak, and hypercharge generators acting on $\mathcal{F}$. The embedding tensors ${E^A}_{(s)}$ (with $s \in \{(3)a, (2)i, (1)\}$) select the corresponding directions in $\mathfrak{g}$:

$$T^a_{(3)} \;=\; {E^A}_{(3)a}\, T_A, \qquad T^i_{(2)} \;=\; {E^A}_{(2)i}\, T_A, \qquad Y \;=\; n_Y\, {E^A}_{(1)}\, T_A, \tag{15.2}$$

where $n_Y > 0$ is the *hypercharge normalization* fixed below.

Hypercharge normalization and $U(1)_Y$.

We adopt the physics normalization for nonabelian factors $\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$ on each irrep. For the abelian factor we introduce n_Y by

$$Y_{\text{canon}} \;=\; n_Y\, Y_0, \qquad g_1 \;=\; g_Y/n_Y, \tag{15.3}$$

with two standard choices: (i) SM normalization $n_Y = 1$; (ii) GUT normalization $n_Y = \sqrt{5/3}$. The same n_Y must be used consistently in the YM–anchor (§5) so that the relation $g_1^{-2} = \kappa_G\, h_{AB} E^A{}_{(1)} E^B{}_{(1)}$ holds in canonical units.

Electric charge.

On each multiplet, the electric charge operator is

$$Q \;=\; T^3_{(2)} + Y_{\text{canon}}. \tag{15.4}$$

Eigenvalues q_α of Q determine the physical charge (in units of e). With $n_Y = \sqrt{5/3}$, the usual SM spectrum (in units of e) is recovered.

Canonical SM matter set (per generation).

For reference, the standard assignments read

Field	$SU(3)_c$	$SU(2)_L$	Y_{canon}
Q_L	**3**	**2**	$+1/6$
u_R	**3**	**1**	$+2/3$
d_R	**3**	**1**	$-1/3$
L_L	**1**	**2**	$-1/2$
e_R	**1**	**1**	-1
ν_R (optional)	**1**	**1**	0
Higgs H	**1**	**2**	$+1/2$

In the USFT dictionary these arise as a subset of $\{(R_3^{(\alpha)}, R_2^{(\alpha)})_{y_\alpha}\}$. Additional vectorlike pairs are allowed but must be flagged for anomaly and mass audits (§15.2 and §15.3).

Programmatic checks (local).

For each multiplet α we compute

$$y_\alpha \;=\; \frac{1}{d_\alpha}\,\mathrm{Tr}\Big(P^{(\alpha)} Y_{\text{canon}}\Big), \qquad c_2^{(2)}(\alpha) = \frac{1}{d_\alpha}\mathrm{Tr}\Big(P^{(\alpha)}\, T^i_{(2)} T^i_{(2)}\Big), \quad c_2^{(3)}(\alpha) = \frac{1}{d_\alpha}\mathrm{Tr}\Big(P^{(\alpha)}\, T^a_{(3)} T^a_{(3)}\Big),$$

with $d_\alpha = \mathrm{rank}\, P^{(\alpha)}$. These invariants feed the RG tables and the anomaly sums (next section). Electric charges are verified by diagonalizing Q on each doublet/singlet.

HDF5 keys (assignments and normalization).

Extend `/sm_map` with:

- `assign/labels` (string[N_{mult}]): names `"Q_L"`, `"u_R"`, ...;
- `assign/R3_dim`, `assign/R2_dim` (int32[N_{mult}]);
- `assign/Y` (float64[N_{mult}]): hypercharges y_α;
- `assign/mult` (int32[N_{mult}]): multiplicities $\mathcal{N}_\alpha$;
- `assign/Q_spec` (float64[N_{mult}, $d_{\max}$]): charge spectra of Q;
- `P_proj` (sparse or block indices): projectors $P^{(\alpha)}$;
- `nY_convention` (string): `"SM"` or `"GUT(5/3)"` (must match YM–anchor);
- `generations` (int32): $\mathcal{N}_u, \mathcal{N}_d, \mathcal{N}_\ell$.

All datasets carry `units` and `axes`. Integrity tests ensure that $Q = T^3 + Y_{\text{canon}}$ spectra match the tabulated `assign/Q_spec` and that the chosen `nY_convention` aligns with the reported g_1 in `/sm_map/g123_ref`.

Acceptance (local, before anomalies).

Acceptance 15.1. SM dictionary (local)

- **Normalization consistency.** The same n_Y is used across the YM–anchor and dictionary; g_1 from `/sm_map/g123_ref` matches the abelian kinetic normalization within tolerance.
- **Charge audit.** $Q = T^3 + Y_{\text{canon}}$ eigenvalues on each multiplet equal the reported `assign/Q_spec`; for the canonical set they are $\{\frac{2}{3}, -\frac{1}{3}\}$ on quark doublets and $\{0, -1\}$ on lepton doublets.
- **Record completeness.** `/sm_map/assign/*`, `P_proj`, and `nY_convention` exist with units and pass schema unit tests.

Remark (toward anomalies and RG).

With the dictionary fixed, the anomaly sums $[SU(3)]^2U(1)$, $[SU(2)]^2U(1)$, $[U(1)]^3$, and grav$^2-U(1)$, together with RG continuity to M_Z, are evaluated in §15.2 and §15.3.

15.2 Anomaly cancellation checks

Setup and conventions (left–Weyl basis).

All *chiral* gauge anomalies are computed using left–handed Weyl fermions. Right–handed fermions are converted to their left–handed charge–conjugate fields, which flips abelian charges and conjugates nonabelian representations. Scalars and vectorlike pairs do not

contribute. Generator traces are taken in the physics convention $\mathrm{Tr}(T^aT^b) = \frac{1}{2}\,\delta^{ab}$ on each irrep.

Local (perturbative) gauge anomalies.

For a multiplet α with multiplicity $\mathcal{N}_\alpha$, color irrep $R_3^{(\alpha)}$ and weak irrep $R_2^{(\alpha)}$, define the Dynkin indices $S_2(R)$ by $\mathrm{Tr}_R(T^aT^b) = S_2(R)\,\delta^{ab}$, and let $d_3^{(\alpha)} = \dim R_3^{(\alpha)}$, $d_2^{(\alpha)} = \dim R_2^{(\alpha)}$. With hypercharge eigenvalue y_α (already in the chosen canonical normalization n_Y, cf. §15.1), the anomaly coefficients are

$$\mathcal{A}_{3^2Y} = \sum_\alpha \mathcal{N}_\alpha\, y_\alpha\, S_2\Big(R_3^{(\alpha)}\Big)\, d_2^{(\alpha)}, \tag{15.5}$$

$$\mathcal{A}_{2^2Y} = \sum_\alpha \mathcal{N}_\alpha\, y_\alpha\, S_2\Big(R_2^{(\alpha)}\Big)\, d_3^{(\alpha)}, \tag{15.6}$$

$$\mathcal{A}_{Y^3} = \sum_\alpha \mathcal{N}_\alpha\, y_\alpha^3\, d_3^{(\alpha)} d_2^{(\alpha)}, \tag{15.7}$$

$$\mathcal{A}_{\mathrm{grav}^2Y} = \sum_\alpha \mathcal{N}_\alpha\, y_\alpha\, d_3^{(\alpha)} d_2^{(\alpha)}. \tag{15.8}$$

Cancellation conditions:

$$\boxed{\mathcal{A}_{3^2Y} = \mathcal{A}_{2^2Y} = \mathcal{A}_{Y^3} = \mathcal{A}_{\mathrm{grav}^2Y} = 0}.$$

(Nonabelian cubic local anomalies $\mathcal{A}_{3^3}$ and $\mathcal{A}_{2^3}$ vanish in the SM because $d^{abc} = 0$ for $SU(2)$ irreps and the SM color content is vectorlike with respect to $SU(3)_c$.)

Global (Witten) $SU(2)$ anomaly.

Beyond perturbative anomalies, $SU(2)_L$ admits a *global* anomaly unless the total number of left–handed doublets is even (counting color multiplicity). Let

$$N_{\mathrm{doublets}} := \sum_{\alpha:\ R_2^{(\alpha)}=\mathbf{2}} \mathcal{N}_\alpha\, d_3^{(\alpha)}.$$

Cancellation condition: $\boxed{N_{\mathrm{doublets}} \equiv 0 \pmod 2}$.

Canonical SM per–generation check (sanity).

With one family in the left–Weyl basis $\{Q_L(3,2,+\frac{1}{6}),\ u_R^c(\bar{3},1,-\frac{2}{3}),\ d_R^c(\bar{3},1,+\frac{1}{3}),\ L_L(1,2,-\frac{1}{2}),\ e_R^c(1,1,+1)$

$$\mathcal{A}_{3^2Y}:\ \tfrac{1}{6}\cdot S_2(3)\cdot 2 + \Big(-\tfrac{2}{3}\Big)\cdot S_2(\bar{3})\cdot 1 + \Big(+\tfrac{1}{3}\Big)\cdot S_2(\bar{3})\cdot 1 = \tfrac{1}{6}\cdot\tfrac{1}{2}\cdot 2 - \tfrac{2}{3}\cdot\tfrac{1}{2} + \tfrac{1}{3}\cdot\tfrac{1}{2} = 0,$$

$$\mathcal{A}_{2^2Y}:\ \tfrac{1}{6}\cdot S_2(2)\cdot 3 + \Big(-\tfrac{1}{2}\Big)\cdot S_2(2)\cdot 1 = \tfrac{1}{6}\cdot\tfrac{1}{2}\cdot 3 - \tfrac{1}{2}\cdot\tfrac{1}{2} = 0,$$

$$\mathcal{A}_{Y^3}:\ 3\cdot 2\cdot\Big(\tfrac{1}{6}\Big)^3 + 3\cdot\Big(-\tfrac{2}{3}\Big)^3 + 3\cdot\Big(+\tfrac{1}{3}\Big)^3 + 2\cdot\Big(-\tfrac{1}{2}\Big)^3 + \Big(+1\Big)^3 = 0,$$

$$\mathcal{A}_{\mathrm{grav}^2Y}:\ 3\cdot 2\cdot\tfrac{1}{6} + 3\cdot\Big(-\tfrac{2}{3}\Big) + 3\cdot\Big(+\tfrac{1}{3}\Big) + 2\cdot\Big(-\tfrac{1}{2}\Big) + \Big(+1\Big) = 0,$$

and $N_{\text{doublets}} = 3 + 1 = 4$ is even. Hence each generation is anomaly–free. (Adding a sterile ν_R with $Y = 0$ does not affect the sums.)

Vectorlike extensions and decoupling.

If $\mathcal{F}$ contains vectorlike pairs (ψ_L, ψ_R) in conjugate representations with equal $|Y|$, their net contribution to (15.5)–(15.8) cancels. Such pairs can be given large Dirac masses and decoupled without spoiling anomaly freedom; they must be flagged in the dictionary for RG thresholds (§15.3).

Optional inflow (Green–Schwarz) hooks.

If the raw sums yield nonzero anomaly coefficients, the model must supply a compensating inflow (e.g. axion–like fields with Chern–Simons couplings). We expose hooks *only* for audit: by default USFT enforces strict cancellation without inflow. If enabled, store counterterm coefficients $k_{3^2Y}, k_{2^2Y}, k_{Y^3}, k_{\text{grav}^2Y}$ and require $\mathcal{A}_\bullet + k_\bullet = 0$.

HDF5 keys (anomaly pack).

Under `/sm_map/anomaly`:

- `A_33Y`, `A_22Y`, `A_Y3`, `A_gravY` (float64): totals of (15.5)–(15.8).
- `contrib` (table): per–multiplet contributions with columns `label`, `d3`, `d2`, `y`, `N`, and the four partials.
- `N_doublets` (int32): N_{doublets} for the Witten test.
- `global_SU2_ok` (uint8): 1 if N_{doublets} even, else 0.
- `inflow_enabled` (uint8), `k_33Y`, `k_22Y`, `k_Y3`, `k_gravY` (float64): optional counterterms.

All entries carry `units` (dimensionless) and are hashed in `/audits` with a pass/fail flag.

Acceptance (anomaly gate).

Acceptance 15.2. Anomaly cancellation A model passes iff

1. $\mathcal{A}_{3^2Y} = \mathcal{A}_{2^2Y} = \mathcal{A}_{Y^3} = \mathcal{A}_{\text{grav}^2Y} = 0$ within numerical tolerance (or exactly zero in rational arithmetic);
2. N_{doublets} is even (`global_SU2_ok`=1);
3. the HDF5 anomaly pack exists with per–multiplet breakdown consistent with the dictionary of §15.1;
4. if inflow is enabled, the recorded $k_\bullet$ satisfy $\mathcal{A}_\bullet + k_\bullet = 0$ and the corresponding 2–form couplings are present in `/action/terms`.

15.3 RG continuity to M_Z

Setup and boundary data.

Let $\mu_{\rm ref}$ be the high/reference scale at which the YM–anchor (§5) fixes the *canonical* gauge couplings $g_i(\mu_{\rm ref})$ (with the declared hypercharge normalization n_Y). We evolve $\vec{g} = (g_1, g_2, g_3)$ down to M_Z using an MS$\overline{\rm MS}$ scheme with stepwise decoupling of heavy thresholds. Denote $\alpha_i \equiv g_i^2/(4\pi)$ and $t \equiv \ln \mu$.

One– and two–loop RGEs.

The running obeys

$$\frac{d\,\alpha_i}{dt} = \frac{b_i}{2\pi}\,\alpha_i^2 + \frac{1}{8\pi^2}\sum_{j=1}^{3} B_{ij}\,\alpha_i^2\alpha_j - \frac{1}{8\pi^2}\,C_i^t\,\alpha_i^2\,y_t^2 \;+\; \cdots\,, \tag{15.9}$$

where y_t is the (running) top Yukawa (dominant two–loop Yukawa effect). For the *minimal SM* (one Higgs doublet) in GUT hypercharge normalization $n_Y = \sqrt{5/3}$ the standard coefficients are

$$\vec{b} \;=\; \left(\tfrac{41}{10},\, -\tfrac{19}{6},\, -\,7\right), \qquad B \;=\; \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix}, \qquad \vec{C}^{\,t} \;=\; \left(\tfrac{17}{10},\, \tfrac{3}{2},\, 2\right). \tag{15.10}$$

If the *SM* normalization $n_Y = 1$ is used instead, the abelian entries rescale consistently (e.g. $b_1 = \frac{41}{6}$); in all cases the chosen n_Y is recorded under `/sm_map/nY_convention` and must match the YM–anchor and dictionary (§15.1).

Thresholds and decoupling.

Let $\{\mu_k\}$ be heavy mass thresholds (vectorlike pairs, extra scalars, etc.). We employ step decoupling with MS$\overline{\rm MS}$ matching:

$$\alpha_i^{(\rm below)}(\mu_k) = \alpha_i^{(\rm above)}(\mu_k)\left[1 - \tfrac{\alpha_i}{6\pi}\,\Delta b_i^{(k)}\,\ln(\mu_k^2/m_k^2)\right] \;+\; O(\alpha_i^3), \tag{15.11}$$

where $\Delta b_i^{(k)}$ is the one–loop contribution of species k to b_i, and m_k its pole mass. Between thresholds, evolve with (15.9) using the appropriate effective (b_i, B_{ij}, C_i^t). All thresholds and scheme tags are written under `/rg/flows`.

Closed one–loop solution (for diagnostics).

For coarse checks,

$$\alpha_i^{-1}(\mu) \;=\; \alpha_i^{-1}(\mu_0) \;-\; \frac{b_i}{2\pi}\,\ln\frac{\mu}{\mu_0} \;-\; \sum_{\mu_k\in(\mu,\mu_0)} \frac{\Delta b_i^{(k)}}{2\pi}\,\ln\frac{\mu_k}{\mu_{k-1}} \;+\; O(\alpha). \tag{15.12}$$

We require numerical integrators to reproduce (15.12) within a declared tolerance when two–loop/Yukawa terms are disabled.

Matching at M_Z and observables.

At $\mu = M_Z$ we form the predicted weak angle and strong coupling:

$$\sin^2\theta_W(M_Z) \;=\; \frac{g_1^2}{g_1^2+g_2^2}\bigg|_{M_Z}, \qquad \alpha_s(M_Z) \;=\; \frac{g_3^2(M_Z)}{4\pi}. \tag{15.13}$$

If desired, one may cross–check against the EM coupling via $\alpha_{\rm EM}^{-1}(M_Z) = \alpha_1^{-1}(M_Z) + \alpha_2^{-1}(M_Z)$ (in GUT normalization), including standard EW scheme conversions.

HDF5 keys (RG pack).

Under `/rg/flows` we store:

- `mu` (float64[n_μ], units: energy), `g123` (float64[n_μ,3]).
- `b` (float64[3]), `B` (float64[3,3]), `C_yt` (float64[3]); attributes: `scheme="MSbar"`, `nY_convention`.
- `thresholds` (table): rows {`label`, `m_pole`, `Delta_b1,2,3`}.
- `MZ_pred` (group): `sin2thetaW`, `alpha3`, `g1,g2,g3` at M_Z.
- `ref` (group): `mu_ref`, `g123_ref` (from YM–anchor).

Consistency checks enforce monotone μ grids, proper units, and that the integrated (g_i) satisfy the piecewise one–loop relation (15.12) within tolerance when requested.

Acceptance (RG continuity).

A run passes the RG continuity gate if:

1. The numerical integrator with recorded `scheme` and `thresholds` reproduces the stored `g123` to within a relative tolerance $\varepsilon_{\rm rg}$ (CI replay).
2. The predicted $\sin^2\theta_W(M_Z)$ and $\alpha_s(M_Z)$ lie within the declared decision windows (cf. Part VI); or, in a calibration mode, the deviations are reported with uncertainty budgets.
3. The hypercharge convention `nY_convention` matches the SM dictionary and YM–anchor, and the anomaly gate (§15.2) is passed.

Checklist 15.1: SM consistency

- Proper $U(1)_Y$ normalization and charge quantization.
- All gauge/gravitational anomalies cancel.
- RG continuity to M_Z with stated scheme.

Part VI

Signature Portfolio and Kill Windows

Chapter 16

Cosmology Signatures

Prelude. On cosmological backgrounds, the spectral medium inherits an effective FRW geometry from g_{eff} and a hierarchy of relaxation scales from the descriptors $(\rho, \eta, \chi^\mu, T)$. This chapter converts those ingredients into observables: the effective equation of state $w_{\text{eff}}(a)$, anisotropy templates, and pipelines that attach *kill windows* to CMB/LSS comparisons. We begin with $w_{\text{eff}}(a)$ as driven by the *η-relaxation* channel and its correlations with the ρ-sector.

16.1 $w_{\text{eff}}(a)$ from η-relaxation; correlations with ρ-field

Background setup.

Work on a spatially flat FRW slice of the emergent metric g_{eff},

$$ds^2_{\text{eff}} \;=\; -N^2(\tau)\, d\tau^2 \;+\; a^2(\tau)\, \delta_{ij}\, dx^i dx^j, \qquad \mathcal{H} \equiv \frac{a'}{a}, \quad H \equiv \frac{\dot{a}}{a},$$

with conformal (prime) or cosmic (dot) time as convenient and lapse N (often set to 1). Let $T^\mu{}_\nu$ denote the *total* stress tensor sourced by the spectral action $\mathcal{L}[\Phi, \rho, \eta, \chi^\mu, T]$ after integrating out fast gauge/matter modes at scales $\gg H$. Define

$$\rho_{\text{eff}} \;\equiv\; \langle T^0{}_0 \rangle_{\text{bg}}\,(-1), \qquad p_{\text{eff}} \;\equiv\; \tfrac{1}{3}\,\langle T^i{}_i \rangle_{\text{bg}}, \qquad w_{\text{eff}}(a) \;\equiv\; \frac{p_{\text{eff}}}{\rho_{\text{eff}}},$$

where $\langle \cdot \rangle_{\text{bg}}$ is the spatially homogeneous expectation on the background sector. The cosmological evolution satisfies

$$3M^2_{\text{Pl,eff}}(a)\, H^2 \;=\; \rho_{\text{eff}}, \qquad -2M^2_{\text{Pl,eff}}(a)\, \dot{H} \;=\; \rho_{\text{eff}} + p_{\text{eff}}, \tag{16.1}$$

with $M^2_{\text{Pl,eff}}(a)$ the (generally weakly varying) effective Planck mass generated by the geometric block in $\mathcal{L}$.

η-channel and decomposition.

Split the total energy and pressure into a quasi-adiabatic baseline (ΛCDM fields mapped through the dictionary) plus spectral corrections from (η, ρ):

$$\rho_{\text{eff}} = \rho_{\Lambda\text{CDM}} \;+\; \rho_\eta \;+\; \rho_\rho \;+\; \rho_{\text{int}}, \tag{16.2}$$

$$p_{\text{eff}} = p_{\Lambda\text{CDM}} \;+\; p_\eta \;+\; p_\rho \;+\; p_{\text{int}}. \tag{16.3}$$

Here $\rho_{\text{int}}, p_{\text{int}}$ collect bilinear mixings fixed by the quadratic ($\mathcal{L}^{(2)}$) and cubic terms retained on the background. For a canonical η-sector with relaxation potential $U(\eta)$ and (optional) non-minimal coupling $\xi_\eta(\eta)\,\mathcal{R}_{\text{eff}}$,

$$\rho_\eta = \tfrac{1}{2}\dot{\eta}^2 \;+\; U(\eta) \;+\; 3\xi_\eta H^2\eta^2 \;+\; \cdots, \tag{16.4}$$

$$p_\eta = \tfrac{1}{2}\dot{\eta}^2 \;-\; U(\eta) \;-\; \xi_\eta\Big(2\dot{H} + 3H^2\Big)\eta^2 \;+\; \cdots, \tag{16.5}$$

with ellipses for controlled higher-derivative operators inside the declared DoV.

Relaxation dynamics (operational model).

Linearize η about a slowly drifting minimum $\bar{\eta}(a)$ and include the colored, CPTP-consistent damping kernel Γ_η induced by the GKSL channels:

$$\ddot{\eta} + 3H\dot{\eta} \;+\; m_\eta^2(a)\,\Big(\eta - \bar{\eta}(a)\Big) \;+\; \underbrace{\int_{t_0}^{t} K_\eta(t-t')\,\dot{\eta}(t')\,dt'}_{\text{colored dissipation}} \;=\; J_\eta[\rho] \;+\; \xi_\eta(t), \tag{16.6}$$

where $m_\eta^2 \equiv U''(\bar{\eta})$, K_η admits a CP time-local embedding (§12), $J_\eta[\rho]$ is the ρ-sector source (see below), and ξ_η is a zero-mean noise consistent with fluctuation–dissipation on the coarse-grained slice. In the Markovian limit $K_\eta \to 2\gamma_\eta\,\delta$, (16.6) reduces to $\ddot{\eta} + 3H\dot{\eta} + \gamma_\eta\dot{\eta} + m_\eta^2(\eta - \bar{\eta}) = J_\eta + \xi_\eta$.

ρ-sector and cross-coupling.

Treat ρ as the medium's *spectral density* field controlling stiffness of gauge/metric blocks; in the background, write a minimal bilinear coupling (consistent with the quadratic action and symmetries)

$$\mathcal{L}_{\text{int}}^{(\eta\rho)} \;=\; -\,\lambda_{\eta\rho}(a)\,\Big(\eta - \bar{\eta}\Big)\,\Big(\rho - \bar{\rho}\Big), \qquad J_\eta[\rho] \;=\; \lambda_{\eta\rho}(a)\,\Big(\rho - \bar{\rho}\Big), \tag{16.7}$$

with $\bar{\rho}(a)$ the adiabatic track. On the background, ρ may itself follow a relaxation $\dot{\rho} + \Gamma_\rho(\rho - \bar{\rho}) = \xi_\rho$; the joint (η, ρ) system then yields correlated pressure and energy corrections.

Effective equation of state.

Define the *spectral fraction* $\Omega_{\rm spec}(a) \equiv \Big(\rho_\eta + \rho_\rho + \rho_{\rm int}\Big)/\rho_{\rm eff}$ and its EoS $w_{\rm spec} \equiv (p_\eta + p_\rho + p_{\rm int})/(\rho_\eta + \rho_\rho + \rho_{\rm int})$. Then

$$w_{\rm eff}(a) \;=\; \Big(1 - \Omega_{\rm spec}\Big)\, w_{\Lambda\rm CDM}(a) \;+\; \Omega_{\rm spec}(a)\, w_{\rm spec}(a), \tag{16.8}$$

with

$$w_{\rm spec}(a) \;=\; \frac{\frac{1}{2}\dot\eta^2 - U(\eta) - \xi_\eta(2\dot H + 3H^2)\eta^2 + p_\rho + p_{\rm int}}{\frac{1}{2}\dot\eta^2 + U(\eta) + 3\xi_\eta H^2\eta^2 + \rho_\rho + \rho_{\rm int}}. \tag{16.9}$$

In the *slow–roll/overdamped* regime of (16.6) ($\dot\eta \simeq 0$ and $|\ddot\eta| \ll H|\dot\eta|$), $w_{\rm spec} \approx -1$ up to mixing terms; away from the minimum, the kinetic piece raises $w_{\rm spec}$ towards 0 (matter-like) or +1 (stiff) depending on Γ_η and m_η/H.

Correlation diagnostics.

Define the background cross-covariance and correlation coefficient

$$\mathrm{Cov}_{\eta\rho}(a) \;\equiv\; \langle \delta\eta\, \delta\rho\rangle_{\rm bg}, \qquad r_{\eta\rho}(a) \;\equiv\; \frac{\mathrm{Cov}_{\eta\rho}}{\sigma_\eta\, \sigma_\rho},$$

and the induced response of $w_{\rm eff}$,

$$\delta w_{\rm eff}(a) \;\simeq\; \frac{\partial w_{\rm eff}}{\partial \eta}\,\delta\eta \;+\; \frac{\partial w_{\rm eff}}{\partial \rho}\,\delta\rho \;+\; \cdots, \tag{16.10}$$

so that $\mathrm{Var}\Big[w_{\rm eff}(a)\Big]$ picks a correlated piece $2(\partial_\eta w_{\rm eff})(\partial_\rho w_{\rm eff})\,\mathrm{Cov}_{\eta\rho}$. At the perturbation level, record the large–scale cross–spectrum $P_{\eta\rho}(k,a) = \langle \delta\eta_{\boldsymbol{k}}\, \delta\rho_{-\boldsymbol{k}}\rangle$ for CMB/LSS pipelines.

Compact parametric form (forecast layer).

For forecasting and comparisons, it is useful to expose a calibrated two-parameter family (valid on the declared DoV):

$$w_{\rm eff}(a) \;=\; w_0 \;+\; (1-a)\, w_a \;+\; \Delta w_{\rm rel}(a;\tau_\eta,\lambda_{\eta\rho}) \;+\; O\Big((\rho_{\rm int}/\rho_{\rm eff})^2\Big), \tag{16.11}$$

where w_0, w_a are the CPL baseline and $\Delta w_{\rm rel}$ captures the *relaxation shoulder* controlled by $\tau_\eta \equiv (3H + \gamma_\eta)^{-1}$ and the coupling $\lambda_{\eta\rho}$. All three are fitted to the full background solution of (16.6)–(16.7).

Acceptance and kill windows.

A cosmology run passes this section's gate if: (i) the background energy condition $\rho_{\rm eff} > 0$ holds and $|M'_{\rm Pl,eff}|/M_{\rm Pl,eff} \ll 1$ on the DoV; (ii) the calibrated $w_{\rm eff}(a)$ from the solver matches the parametric surrogate (16.11) within a declared tolerance; (iii) the resulting CMB/LSS likelihood lies within the published *kill window* for w_0, w_a (and any extended knobs), with the HDF5 predictions written under `/predictions/weff` and cross–spectra

under `/backgrounds/frw/profiles`.

HDF5 keys.

Store under:

- `/backgrounds/frw/params`: $\{w_0, w_a, \tau_\eta, \lambda_{\eta\rho}, \xi_\eta\}$ with units.
- `/backgrounds/frw/profiles`: arrays of a, $H(a)$, $\rho_{\rm eff}(a)$, $p_{\rm eff}(a)$, $w_{\rm eff}(a)$, and $P_{\eta\rho}(k,a)$ (axes: $[a]$ or $[a,k]$).
- `/predictions/weff`: compressed spline/control points for $w_{\rm eff}(a)$ and the surrogate parameters, plus covariance.

All datasets carry `units` and `axes` attributes and are included in the G/D audits.

16.2 Statistical anisotropy: $g_*(k)$, $c_s(\theta)$, bispectrum templates

Physical origin.

A homogeneous background thread χ^μ in the spectral medium selects a preferred spatial direction $\hat{\chi}$ on FRW slices of $g_{\rm eff}$. Quadratic perturbations of the relevant scalar sector Q (adiabatic mode) and of vector/tensor sectors inherit this anisotropy through the kinetic/gradient blocks and through GKSL-induced colored damping. We parameterize the leading (weak) anisotropy by a quadrupolar deformation aligned with $\hat{\chi}$ and propagate it consistently to power spectra, sound cones, and bispectra.

Quadrupolar power spectrum and $g_*(k)$.

Let ζ denote the comoving curvature perturbation reconstructed from the spectral variables (dictionary fixed in Part V). For wavevector $\boldsymbol{k}$ and $\hat{\boldsymbol{k}} \equiv \boldsymbol{k}/k$, define

$$P_\zeta(\boldsymbol{k}) \;=\; P_0(k)\Big[1 + g_*(k)\,\mathcal{Q}_2(\hat{\boldsymbol{k}}\cdot\hat{\chi}) \;+\; O\Big(g_*^2\Big)\Big], \qquad \mathcal{Q}_2(x) \;\equiv\; \frac{3x^2-1}{2}, \tag{16.12}$$

where $P_0(k)$ is the isotropic baseline. The *spectral prediction* for $g_*(k)$ follows from the quadratic action of ζ:

$$S^{(2)}[\zeta] = \frac{1}{2}\int d\tau\, d^3x\, a^2\Big[\mathsf{K}_{ij}\,\partial_\tau\zeta_i\,\partial_\tau\zeta_j - \mathsf{Z}^{mn}_{ij}\,\partial_m\zeta_i\,\partial_n\zeta_j \;+\; \dots\Big], \tag{16.13}$$

with indices i,j labeling coupled fluid/field components entering ζ and (m,n) spatial directions. In the single-light-mode limit, the angular dependence of the phase/Group speeds yields

$$g_*(k) \;\simeq\; \alpha_2(k)\,\epsilon_\chi(a_k) \quad \text{with} \quad \epsilon_\chi \equiv \frac{\hat{\chi}^m\hat{\chi}^n\,\Delta\mathsf{Z}_{mn}}{\mathrm{tr}(\mathsf{Z})} \;-\; \frac{\hat{\chi}^m\hat{\chi}^n\,\Delta\mathsf{K}_{mn}}{\mathrm{tr}(\mathsf{K})}, \tag{16.14}$$

where $\Delta\mathsf{Z}_{mn}$ and $\Delta\mathsf{K}_{mn}$ are the anisotropic parts induced by the χ-sector, a_k the horizon-crossing scale for mode k, and $\alpha_2(k)$ a calculable transfer coefficient (unity in the simplest slow-roll, weak-mixing limit). Equation (16.14) ties g_* directly to geometric entries of the quadratic operator and thus to the Lie–metric and medium overlaps fixed earlier.

Angular sound speed $c_s(\theta)$.

Write the direction-dependent sound cone as

$$c_s^2(\theta) \;=\; c_{s0}^2\left[1 \;+\; \delta_c(a)\,\mathcal{Q}_2\Big(\cos\theta\Big) \;+\; O(\delta_c^2)\right], \qquad \cos\theta \equiv \hat{\boldsymbol{k}}\cdot\hat{\chi}, \tag{16.15}$$

with baseline $c_{s0}^2 = \mathrm{tr}(\mathsf{Z})/\mathrm{tr}(\mathsf{K})$. At leading order, using the principal-symbol analysis from Part III and the LMI cone certificates,

$$\delta_c(a) \;\simeq\; \frac{3}{2}\left[\frac{\hat{\chi}^m\hat{\chi}^n\,\Delta\mathsf{Z}_{mn}}{\mathrm{tr}(\mathsf{Z})} \;-\; \frac{\hat{\chi}^m\hat{\chi}^n\,\Delta\mathsf{K}_{mn}}{\mathrm{tr}(\mathsf{K})}\right], \qquad \max_\theta \frac{c_s(\theta)}{\mathrm{cEff}} \le 1+\varepsilon_G, \tag{16.16}$$

where the last inequality is the G (cone) audit on FRW slices.

Anisotropic bispectrum templates.

Let $B_\zeta(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$ be defined via $\langle\zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_{123})\,B_\zeta$. The leading quadrupolar templates aligned with $\hat{\chi}$ read

$$B_{\mathrm{loc}}^{(2)} = f_{\mathrm{NL}}^{\parallel}\,\mathcal{Q}_2\Big(\hat{\boldsymbol{k}}_1\cdot\hat{\chi}\Big)\,P_0(k_1)P_0(k_2) \;+\; 2\text{ perms}, \tag{16.17}$$

$$B_{\mathrm{eq}}^{(2)} = f_{\mathrm{NL,eq}}^{\parallel}\left[\mathcal{Q}_2(\hat{\boldsymbol{k}}_1\cdot\hat{\chi}) + \mathcal{Q}_2(\hat{\boldsymbol{k}}_2\cdot\hat{\chi}) + \mathcal{Q}_2(\hat{\boldsymbol{k}}_3\cdot\hat{\chi})\right]\mathcal{S}_{\mathrm{eq}}(k_1,k_2,k_3), \tag{16.18}$$

where $\mathcal{S}_{\mathrm{eq}}$ is the standard equilateral shape. Microscopically, $f_{\mathrm{NL}}^{\parallel}$ scales with the anisotropic cubic vertex coefficients from the χ-deformed action (e.g. from η–Q mixings and anisotropic Z^{mn}). Colored GKSL noise can imprint a scale/running dependence which we parameterize as $f_{\mathrm{NL}}^{\parallel}(k) = f_0\,(k/k_*)^{n_\parallel}$.

Estimators and pipelines.

For CMB or LSS data, adopt optimized KSW-type estimators extended with quadrupolar weights. In harmonic space:

$$\widehat{g}_* = \frac{1}{\mathcal{N}}\sum_{\ell_1 m_1 \ell_2 m_2} a_{\ell_1 m_1} a_{\ell_2 m_2}\,(-1)^{m_2}\,\mathcal{C}^{2M}_{\ell_1 m_1\,\ell_2,-m_2}\,W^{(2)}_{\ell_1\ell_2}, \qquad M=0 \text{ for alignment with } \hat{\chi}, \tag{16.19}$$

with $\mathcal{C}$ a Clebsch–Gordan coefficient and $W^{(2)}$ a transfer kernel precomputed from the Boltzmann solver with direction-dependent $c_s(\theta)$. Bispectrum estimators $\widehat{f}_{\mathrm{NL}}^{\parallel}$ use the corresponding anisotropic separable templates (16.17–16.18).

Consistency ties (theory layer).

The parameters obey consistency relations inherited from the same quadratic blocks:

$$g_*(k) \;\approx\; \beta(k)\,\delta_c(a_k) \quad \text{and} \quad f_{\rm NL}^{\parallel}(k) \;\approx\; \gamma(k)\,\delta_c(a_k)\,\frac{\Lambda_3}{c_{s0}^2}, \tag{16.20}$$

where β, γ are theory-controlled transfer functions and Λ_3 collects the relevant cubic vertices. These relations make the anisotropy portfolio *overconstrained* and suitable for orthogonal-template kill tests.

Acceptance and kill windows.

A cosmology-anisotropy run passes if: (i) cone LMIs confirm (16.16) on the FRW DoV grid (G); (ii) the pipeline returns $(\widehat{g}_*, \widehat{f}_{\rm NL}^{\parallel})$ with variances matching forecasts, and the theory relation (16.20) holds within tolerance; (iii) the pair $(g_*, f_{\rm NL}^{\parallel})$ lies inside the declared windows (`/windows/kill`) or, failing that, the run is flagged as a *kill* with attached decision logs.

HDF5 keys (anisotropy pack).

- `/predictions/g_star`: k–grid and $g_*(k)$; attrs: `axes=[k]`, `units=-`.
- `/predictions/cs_theta`: grid over (a, θ) and $c_s(a, \theta)$; attrs: `axes=[a,theta]`, `units=c`.
- `/predictions/bisp_anis`: compressed coefficients for the templates (16.17–16.18) and $f_{\rm NL}^{\parallel}(k)$ (axes: [k]).
- `/backgrounds/frw/cones`: LMI margins and front-speed ratios for the scalar block ensuring G compliance.
- `/windows/kill/aniso`: bounds for g_*, $f_{\rm NL}^{\parallel}$, and optional slope $n_{\parallel}$ with confidence levels.

Notes.

All quantities above are understood within the declared DoV; any use of higher multipoles ($L \geq 4$) or direction precession of $\hat{\chi}$ must be explicitly enabled and audited in the same HDF5 pack with updated transfer kernels.

16.3 CMB/LSS pipelines; kill windows

Goal.

Turn spectral-medium predictions into *data-ready* angular/3D statistics for CMB and large-scale structure (LSS), then evaluate them against declared *kill windows* with reproducible likelihoods and covariances.

CMB forward model and likelihood

Let the CMB data vector collect pseudo-spectra (after masking) up to a chosen $\ell_{\max}$,

$$\mathbf{d}_{\rm CMB} \;:=\; \Big(\tilde{C}_\ell^{TT}, \tilde{C}_\ell^{TE}, \tilde{C}_\ell^{EE},\; \tilde{C}_L^{\phi\phi}\Big)_{\ell,L} \;\; (+\text{ bispectrum coefficients if enabled}).$$

The *theory* isotropic baseline follows

$$C_\ell^{XY} \;=\; 4\pi \int d\ln k\; \Delta_\ell^X(k)\, \Delta_\ell^Y(k)\, P_\zeta(k), \qquad X, Y \in \{T, E\}, \tag{16.21}$$

with transfer kernels Δ_ℓ^X computed on $g_{\rm eff}$ backgrounds (FRW slices from Part III). Anisotropy from the thread $\hat{\chi}$ enters through $P_\zeta(\boldsymbol{k}) = P_0(k)\,[1 + g_*(k)\,\mathcal{Q}_2(\hat{\boldsymbol{k}}\cdot\hat{\chi})]$ (cf. §16.2), generating BipoSH couplings $A_{\ell\ell'}^{LM}$ with $L = 2$ at leading order. Masks induce mode-mixing

$$\tilde{C}_\ell^{XY} \;=\; \sum_{\ell'} M_{\ell\ell'}^{XY}\, C_{\ell'}^{XY}, \qquad \tilde{A}_{\ell\ell'}^{LM} \;=\; \sum_{\ell_1\ell_2} \mathsf{M}_{\ell\ell',\ell_1\ell_2}^{LM}\, A_{\ell_1\ell_2}^{LM}, \tag{16.22}$$

where M and M are computable from the mask harmonics.

We use a Gaussian (or Wishart-inspired) likelihood for spectra and a Gaussian approximation for BipoSH coefficients:

$$-2\ln \mathcal{L}_{\rm CMB} = \Big(\mathbf{d}_{\rm CMB} - \mathbf{t}_{\rm CMB}\Big)^\top \mathbf{N}_{\rm CMB}^{-1} \Big(\mathbf{d}_{\rm CMB} - \mathbf{t}_{\rm CMB}\Big) \;+\; \ln\!\det \mathbf{N}_{\rm CMB}, \tag{16.23}$$

with $\mathbf{t}_{\rm CMB}$ the masked theory vector (16.21–16.22) and $\mathbf{N}_{\rm CMB}$ the covariance including cosmic variance, beams, noise, and mask couplings. Nuisance parameters (foreground amplitudes, calibrations) are analytically or MCMC marginalized and recorded under `/runs/solver` (YAML) and HDF5 attributes.

LSS forward model (tomographic C_ℓ or 3D $P(k)$)

For tomographic galaxy clustering and lensing we assemble $\mathbf{d}_{\rm LSS} = \{\, \tilde{C}_\ell^{g_i g_j}, \tilde{C}_\ell^{\gamma_i\gamma_j}, \tilde{C}_\ell^{g_i\gamma_j} \,\}$ over redshift bins (i, j), with theory

$$C_\ell^{XY,ij} = 4\pi \int d\ln k\; \mathcal{W}_\ell^{X,i}(k)\, \mathcal{W}_\ell^{Y,j}(k)\, P_{\rm m}(k, z), \qquad X, Y \in \{g, \gamma\}. \tag{16.24}$$

The kernels $\mathcal{W}_\ell$ incorporate the thread-induced anisotropy via direction-dependent sound cones entering the growth and lensing responses; when the survey strategy averages over orientations, the leading effect is a residual quadrupole that can be captured by anisotropic templates analog to (16.12). Alternatively, for 3D analyses we use $P(k, \mu, z)$ with RSD angle μ and the same anisotropic consistency ties as in §16.2. The LSS likelihood is Gaussian in the binned pseudo-spectra or $P(k, \mu)$ with shot noise and super-sample covariance.

Pipelines, audits, and outputs

- **Forward engines.** Boltzmann + lensing kernels on $g_{\rm eff}$ FRW backgrounds (`/backgrounds/frw`); anisotropic transfer corrections from $c_s(\theta)$ maps (`/predictions/cs_theta`).
- **Masking and mixing.** Compute $M_{\ell\ell'}$, M^{LM} from the exact survey masks; save to `/pipelines/cmb/masks` and `/pipelines/lss/masks`.
- **Covariances.** Semi-analytic CV+noise blocks validated by map simulations (seeded and logged); store as sparse HDF5 datasets with units.
- **Compression.** Optionally compress to bandpowers with linear *lossless* operators recorded as `/pipelines/*/compressor`.

Kill windows and decision rules

For each observable O (power-spectrum band, BipoSH mode, bispectrum amplitude, tomographic combination) define a window

$$\mathcal{W}_O \;=\; \begin{cases} [\,O_{\rm min},\, O_{\rm max}\,] & \text{two-sided,} \\ (-\infty,\, O_{\rm max}] \text{ or } [O_{\rm min}, \infty) & \text{one-sided,} \end{cases}$$

with confidence level stored alongside. Given the theory prediction $\widehat{O}$ and its forecasted uncertainty σ_O (from the same pipeline), form the standardized pull $z_O = (O_{\rm data} - \widehat{O})/\sigma_O$ and compute p_O under the reference distribution. *Decision:* if either $O_{\rm data} \notin \mathcal{W}_O$ *or* $p_O < p_{\rm kill}$ (configurable), flag a *kill* event and write an audit entry with hashes of all inputs.

HDF5 keys (pipelines pack)

- `/pipelines/cmb/theory`: C_ℓ^{XY}, $A_{\ell\ell'}^{LM}$ (if enabled); attrs: `axes` and `units`.
- `/pipelines/cmb/masks`: $M_{\ell\ell'}$, $\mathsf{M}_{\ell\ell'}^{LM}$, mask harmonics, sky fractions.
- `/pipelines/cmb/cov`: block covariance for the data vector; sparse layout.
- `/pipelines/lss/theory`: tomographic $C_\ell^{XY,ij}$ or 3D $P(k,\mu,z)$ tables with kernels $\mathcal{W}$.
- `/pipelines/lss/cov`: survey-specific covariances (shot noise, SSC).
- `/windows/kill/cmb`, `/windows/kill/lss`: JSON-like tables of windows per observable with `bound_type`, `lower`, `upper`, `confidence`.
- `/audits/cmb_run`, `/audits/lss_run`: hashes of inputs, masks, seeds; likelihood values; decision logs.

Acceptance.

Predictive intervals (means $\widehat{O}$ with covariances) are published for all released observables; all corresponding kill windows are declared under `/windows/kill`. A pipeline *passes* when (i) likelihoods reproduce validation cases; (ii) covariances match map-based estimates within tolerance; (iii) all decisions (pass/kill) are reproducible from the HDF5 bundle and logs.

Chapter 17

Gravitational-Wave Signatures

17.1 $c_T(f,z)$, attenuation/dispersion from ρ-solitons

Aim.

We derive the frequency– and redshift–dependent tensor speed $c_T(f,z)$ on $(M, g_{\rm eff})$ and quantify amplitude/phase modifications induced by the spectral medium. Inhomogeneous backgrounds sourced by the ρ–sector (solitons, domain filaments) act as a dilute *effective medium* for GWs, producing attenuation and dispersive phase shifts that are calculable from the same quadratic operator and Rayleigh/GKSL blocks fixed in Part III.

Tensor quadratic action and propagation

On FRW slices of $g_{\rm eff}$, let h_{ij} be the transverse–traceless perturbation ($\partial_i h^{ij} = h^i{}_i = 0$). The tensor quadratic action reads

$$S_T^{(2)} = \frac{1}{8}\int d\tau\, d^3x\; a^2\Big[\mathcal{K}_T(\tau,\hat{\boldsymbol{k}})\,(\partial_\tau h_{ij})^2 - \mathcal{Z}_T(\tau,\hat{\boldsymbol{k}})\,(\nabla h_{ij})^2 - a^2 m_T^2(\tau)\,h_{ij}^2\Big] \;+\; S_{\rm diss}[h], \tag{17.1}$$

where the direction dependence (through $\hat{\chi}$) is encoded in the principal blocks $\mathcal{K}_T, \mathcal{Z}_T$ (cf. LMI cones in §10.1.1) and $S_{\rm diss}$ is the time-local colored embedding of the Rayleigh/GKSL dissipation.

In Fourier space ($h_\lambda(\boldsymbol{k},\tau)$, $\lambda = \{+,\times\}$), the evolution equation takes the canonical form

$$h_\lambda'' + \Big[2+\nu_T(\tau,\hat{\boldsymbol{k}})\Big]\mathcal{H}\,h_\lambda' + \Big[c_T^2(\tau,\hat{\boldsymbol{k}})\,k^2 + a^2 m_T^2(\tau)\Big]\,h_\lambda + \gamma_T(\tau,\hat{\boldsymbol{k}})\,h_\lambda' + \delta_T(\tau,\hat{\boldsymbol{k}})\,h_\lambda \;=\; 0, \tag{17.2}$$

with conformal Hubble rate $\mathcal{H} = a'/a$, *tensor speed*

$$c_T^2(\tau,\hat{\boldsymbol{k}}) \;=\; \frac{\mathcal{Z}_T(\tau,\hat{\boldsymbol{k}})}{\mathcal{K}_T(\tau,\hat{\boldsymbol{k}})} \;=\; 1+\delta c_T(\tau,\hat{\boldsymbol{k}}), \qquad \max_{\hat{\boldsymbol{k}}}\frac{c_T}{\rm cEff} \le 1+\varepsilon_G \;\;(\mathsf{G}), \tag{17.3}$$

and friction/masslike corrections γ_T, δ_T inherited from the time-local CP embedding of colored noise (see §??). The dimensionless ν_T captures adiabatic variation of $\mathcal{K}_T$: $\nu_T \equiv \partial_\tau \ln \mathcal{K}_T$.

WKB dispersion and complex self-energy.

For subhorizon modes, a WKB ansatz $h_\lambda \propto \exp\{-i \int \omega \, d\tau\}$ gives

$$\omega^2(k,\tau,\hat{\boldsymbol{k}}) \;=\; c_T^2\, k^2 + a^2 m_T^2 \;+\; \Re\Sigma_T(\omega,k;\tau,\hat{\boldsymbol{k}}), \qquad \alpha_T(\omega,k;\tau,\hat{\boldsymbol{k}}) \;\equiv\; -\frac{\Im\Sigma_T}{\omega} \;\geq 0, \tag{17.4}$$

where Σ_T is the tensor *self-energy* summarizing dissipative and dispersive corrections from the medium. The attenuation coefficient α_T controls amplitude decay; Kramers–Kronig ties $\Re\Sigma_T$ to $\Im\Sigma_T$.

Effective medium from ρ–solitons

Let the ρ–sector host a stationary, dilute population of solitons with comoving number density $n_s(z)$ and differential cross section $d\sigma_T/d\Omega(\omega,\theta)$ for tensor waves. In the single-scattering (Born) regime the transport mean-free path and attenuation rate are

$$\ell_T^{-1}(\omega,z) \;=\; n_s(z)\,\sigma_T(\omega), \qquad \alpha_T(\omega,z) \;=\; \frac{c_T(\omega,z)}{2\,\ell_T(\omega,z)} \;=\; \frac{c_T}{2}\, n_s\, \sigma_T, \tag{17.5}$$

with $\sigma_T(\omega) = \int d\Omega\,(1-\cos\theta)\, d\sigma_T/d\Omega$ the transport cross section. The dispersive shift follows from

$$\Re\Sigma_T(\omega) \;=\; \frac{2}{\pi}\,\mathcal{P}\int_0^\infty d\omega'\,\frac{\omega'\,\Im\Sigma_T(\omega')}{\omega'^2-\omega^2} \;=\; -\frac{2\omega}{\pi}\,\mathcal{P}\int_0^\infty d\omega'\,\frac{\alpha_T(\omega')}{\omega'^2-\omega^2}, \tag{17.6}$$

where $\mathcal{P}$ denotes Cauchy principal value. Microscopically, σ_T is computed from the linearized tensor–soliton interaction derived in the ρ–sector EFT (potential and index-contrast pictures both map to (17.5)); its scaling with ω is recorded with the soliton acceptance pack (cf. §9.3).

Observable mapping: $c_T(f,z)$ and luminosity distance

For a source at redshift z_s observed at frequency $f = \omega/2\pi$, the GW amplitude accumulates a damping factor and phase delay

$$\mathcal{A}(f) \propto \frac{1}{D_L^{\rm gw}(f,z_s)}\,\exp\!\left\{-\int_0^{z_s} dz\,\frac{\alpha_T\big(2\pi f(1+z),z\big)}{H(z)\,(1+z)}\right\}, \tag{17.7}$$

$$\Delta t(f) = \int_0^{z_s} dz\,\frac{1}{H(z)\,(1+z)}\left[\frac{1}{c_T\big(2\pi f(1+z),z\big)} - \frac{1}{\rm cEff}\right]. \tag{17.8}$$

The *GW luminosity distance* $D_L^{\rm gw}$ differs from the electromagnetic one by friction and cone factors:

$$D_L^{\rm gw}(f,z) \;=\; D_L^{\rm em}(z)\,\sqrt{\frac{\rm cEff}{c_T(f,z)}}\;\exp\!\left\{\int_0^z dz'\,\frac{\alpha_T\big(2\pi f(1+z'),z'\big)}{H(z')\,(1+z')}\right\}. \tag{17.9}$$

Equations (17.7)–(17.9) define the data-facing *GW signature pack*, jointly constrained by standard sirens, binary dispersion tests, and stochastic-background propagation.

Consistency and gates.

(i) **Cone gate** (G): verify $c_T \le \mathrm{cEff}(1+\varepsilon_G)$ across the FRW DoV grid using the tensor principal symbol (stored under `/backgrounds/frw/cones`). (ii) **Positivity** (S): the spectral density implied by $\alpha_T \ge 0$ must produce a nonnegative dissipation rate in the Rayleigh ledger. (iii) **CP** (TQ): the colored embedding used for γ_T, δ_T must be CPTP at the channel level (Choi margins).

HDF5 keys (GW pack)

- `/predictions/gw/cT`: table on (f,z) for $c_T(f,z)$; attrs: `axes=[f,z]`, `units=c`.
- `/predictions/gw/alphaT`: (f,z) grid for $\alpha_T(f,z)$; units: 1/s.
- `/predictions/gw/dispersion`: (f,z) grid for $\Re\Sigma_T/\omega^2$ or phase delay $\Delta t(f,z)$; units: `s`.
- `/backgrounds/frw/cones/tensor`: LMI margins and $\max(c_T/\mathrm{cEff})$ per slice.
- `/windows/kill/gw`: bounds for $(c_T-1)(f,z)$ and for integrated damping $\int \alpha_T/H(1+z)$ with confidence levels.
- `/solitons`: $n_s(z)$ and $\sigma_T(f)$ used in (17.5); units attached; provenance hash.

Acceptance (GW).

A GW release passes when: (a) cone LMIs certify (17.3) on the FRW DoV; (b) $c_T(f,z)$ and $\alpha_T(f,z)$ tables are monotone-continuous on their grids and reproduce the WKB relations (17.4)–(17.6) within tolerance; (c) the pair $\big(D_L^{\rm gw}(f,z),\Delta t(f)\big)$ lies inside declared `/windows/kill/gw` or triggers an auditable kill decision.

17.2 Kill tests with one-sided bounds

Purpose.

We define decision rules that *kill* the spectral–GW sector when either the tensor speed departs upward from the causal cone or the integrated damping is too large. Both are

encoded as *one-sided* hypothesis tests with predeclared bounds (windows) and systematics budgets recorded in HDF5 (see keys below).

One-sided bounds and null hypotheses

Let the reference windows be tables on (f, z):

$$U_{c_T}(f, z) \geq 0, \qquad U_\alpha(f, z) \geq 0,$$

interpreted as (i) an *upper* bound on the superluminal excess $\delta c_T(f, z) := c_T(f, z)/\mathrm{cEff} - 1$ and (ii) an *upper* bound on the redshift–integrated damping,

$$\mathcal{A}(f; z_s) \;:=\; \int_0^{z_s} \frac{\alpha_T\Big(2\pi f(1+z), z\Big)}{H(z)\,(1+z)}\, dz.$$

The catalog-level *nulls* are

$$\mathrm{H0}_{c_T} :\; \delta c_T(f, z) \leq U_{c_T}(f, z), \quad \mathrm{H0}_\alpha :\; \mathcal{A}(f; z_s) \leq U_\alpha(f; z_s). \tag{17.10}$$

Violations of either null at stated confidence trigger a kill.

Per-event test statistics

For an event e with posterior samples for $(\delta c_T, \mathcal{A})$, define one-sided z-scores (Gaussian proxy) or profile-likelihood ratios:

$$Z_{c_T}^{(e)}(f, z) := \frac{\widehat{\delta c_T}^{(e)}(f, z) - U_{c_T}(f, z)}{\sigma_{c_T}^{(e)}(f, z)}, \qquad T_{c_T}^{(e)} := -2 \log \frac{\mathcal{L}(\delta c_T = U_{c_T})}{\max_{\delta c_T} \mathcal{L}(\delta c_T)}, \tag{17.11}$$

$$Z_\alpha^{(e)}(f; z_s) := \frac{\widehat{\mathcal{A}}^{(e)}(f; z_s) - U_\alpha(f; z_s)}{\sigma_{\mathcal{A}}^{(e)}(f; z_s)}, \qquad T_\alpha^{(e)} := -2 \log \frac{\mathcal{L}(\mathcal{A} = U_\alpha)}{\max_{\mathcal{A}} \mathcal{L}(\mathcal{A})}. \tag{17.12}$$

Here hats denote posterior means (or MAP estimates) and σ the posterior standard deviations from the event inference. In the large-sample limit, T approximately follows a χ_1^2 tail (one-sided) under Wilks; otherwise, calibrate by injection (below).

Catalog combination and decision rule

Aggregate across a catalog $\mathcal{C}$ of events (or frequency bins):

$$T_{c_T}^{\mathrm{cat}} := \sum_{e \in \mathcal{C}} w_e\, T_{c_T}^{(e)}, \qquad T_\alpha^{\mathrm{cat}} := \sum_{e \in \mathcal{C}} w_e\, T_\alpha^{(e)}, \qquad \sum_e w_e = 1, \tag{17.13}$$

with inverse-variance or SNR-based weights w_e (stored in HDF5). Define p-values under the nulls in (17.10) either by the χ^2 tails (asymptotic) or by the empirical CDF from calibrated injections:

$$p_{c_T} = \Pr\Big[T_{c_T}^{\text{cat, null}} \geq T_{c_T}^{\text{cat,obs}}\Big], \qquad p_\alpha = \Pr\Big[T_\alpha^{\text{cat, null}} \geq T_\alpha^{\text{cat,obs}}\Big].$$

Kill decision:

$$\text{Kill if} \quad p_{c_T} < p_{\text{gate}} \quad \text{or} \quad p_\alpha < p_{\text{gate}}, \qquad p_{\text{gate}} \in \{0.05,\, 0.01\} \text{ (declared in manifest).} \tag{17.14}$$

Attach a *decision table* row (gate D) that records which null failed, at what (f, z) or $(f; z_s)$, and the contributing events.

Template-bank mismatch and orthogonal tests

To detect departures that may evade simple c_T/α_T summaries (e.g. phase–frequency *textures*), compare GR vs. USFT waveforms using the noise-weighted inner product

$$(a|b) = 4\,\Re\int_0^\infty \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)}\,df, \qquad \mathcal{M} := 1 - \max_\vartheta \frac{(h_{\text{USFT}}(\vartheta)|h_{\text{GR}})}{\sqrt{(h_{\text{USFT}}|h_{\text{USFT}})(h_{\text{GR}}|h_{\text{GR}})}},$$

with ϑ the USFT nuisance parameters (e.g. α_T shape, weak cone tilt). A one-sided *mismatch* gate rejects the null if $\mathcal{M} > \mathcal{M}_{\max}$ after maximizing over GR parameters (time/phase/masses/spins). Calibrate $\mathcal{M}_{\max}$ by injections at fixed FAR/PPV to control false kills.

Injection tests and confusion matrices (AR)

Build *template banks* for GR and for a grid of USFT (c_T, α_T) deformations. Generate injections with realistic $S_n(f)$, duty cycles, and sky distributions; run the full selection pipeline and compute the confusion matrix

$$\mathbf{C} = \begin{pmatrix} P(\text{GR}\to\text{GR}) & P(\text{USFT}\to\text{GR}) \\ P(\text{GR}\to\text{USFT}) & P(\text{USFT}\to\text{USFT}) \end{pmatrix},$$

as a function of (f, z) and SNR. Store $\mathbf{C}$ and the empirical null distributions of T^{cat} and $\mathcal{M}$; use them to map $T_{\text{obs}}^{\text{cat}}$ to calibrated p-values in (17.14).

Systematics and one-sided margins

Account for calibration and modeling systematics with additive margins:

$$U_{c_T} \;\mapsto\; U_{c_T}^{\text{eff}} = U_{c_T} + \Delta_{c_T}^{\text{sys}}, \qquad U_\alpha \;\mapsto\; U_\alpha^{\text{eff}} = U_\alpha + \Delta_\alpha^{\text{sys}},$$

where Δ^{sys} aggregates strain–calibration, redshift, and population uncertainties. These offsets and their provenance hashes must be declared in the manifest; their values tighten with improved calibration (regression-tested in CI).

HDF5 keys (GW kill pack)

- `/windows/kill/gw/ct_upper`: dataset $U_{c_T}(f, z)$; attrs `axes`=[`f`,`z`], `units`=`dimensionless`.
- `/windows/kill/gw/alpha_upper`: dataset $U_\alpha(f; z_s)$; attrs `axes`=[`f`,`z`], units = 1.
- `/predictions/gw/test/T_ct`, `T_alpha`: observed catalog statistics; attrs include p-values and p_{gate}.
- `/predictions/gw/mismatch`: $\mathcal{M}$ per event; threshold `M_max`.
- `/audits/gw/confusion`: confusion matrices binned in (f, z, SNR).
- `/runs/gw/weights`: w_e and event metadata used in (17.13).

Acceptance (D).

Pass if (i) $p_{c_T} \geq p_{\text{gate}}$ and $p_\alpha \geq p_{\text{gate}}$ after systematic margins; (ii) $\mathcal{M} \leq \mathcal{M}_{\max}$ for all catalog events; (iii) injection-calibrated FAR/PPV meet targets and the confusion matrix diagonals exceed preset minima.

AR.

Template banks; injection tests; confusion matrices; empirical p-value tables and YAML manifests for $U_{c_T}, U_\alpha, \Delta^{\text{sys}}$.

Chapter 18

Particle/Precision Signatures

18.1 (g_1, g_2, g_3) at $\mu_{\rm ref}$; $\sin^2\theta_W$ at M_Z

Aim.

We turn the geometric *YM anchor* into precision, falsifiable observables. First, anchor-derived gauge couplings (g_1, g_2, g_3) are fixed at a chosen reference scale $\mu_{\rm ref}$ from the Lie–metric and embedding tensors. Second, these couplings are transported to experimental scales (e.g. M_Z) by a stated RG scheme with thresholds. The key electroweak observable is $\sin^2\theta_W(M_Z)$, tested against ellipsoidal confidence regions.

From the YM anchor to $(g_1, g_2, g_3)(\mu_{\rm ref})$

Let $E^A{}_{(I)}$ project the spectral connection $\mathcal{A}^A_\mu$ onto the three SM factors ($I = 1, 2, 3$ for $U(1), SU(2), SU(3)$). With the Lie–metric h_{AB} and normalization κ_G, form the kinetic Gram matrix

$$K_{IJ} := \kappa_G\, h_{AB}\, E^A{}_{(I)} E^B{}_{(J)}, \qquad K = K^\top \succ 0.$$

Orthogonally diagonalize $K = O^\top D\, O$ with $D = \mathrm{diag}(\tilde\alpha_1, \tilde\alpha_2, \tilde\alpha_3)$. In the rotated fields the kinetic term is $-\frac{1}{4}\sum_I \tilde\alpha_I F'^I_{\mu\nu} F'^{I\,\mu\nu}$, so the canonically normalized couplings at $\mu_{\rm ref}$ are

$$g_I^{-2}(\mu_{\rm ref}) = \tilde\alpha_I \quad \Longleftrightarrow \quad g_I(\mu_{\rm ref}) = \tilde\alpha_I^{-1/2}. \tag{18.1}$$

Auditability. Store $(h_{AB}, E^A{}_{(I)}, K, O, D)$ and the chosen $U(1)$ normalization n_Y in the HDF5 bundle (keys listed below).

RG transport to M_Z

Choose a renormalization scheme (e.g. $\overline{\rm MS}$) and loop order. The flow obeys

$$\frac{d\, g_i}{d \ln \mu} \;=\; \beta_i(\{g\}, \{y\}, \{\lambda\}) \;=\; \frac{b_i}{16\pi^2}\, g_i^3 \;+\; \text{(two–loop and Yukawa/scalar terms)}, \tag{18.2}$$

with (b_1, b_2, b_3) fixed by the particle content and the $U(1)$ convention. *Consistency note:* if the hypercharge generator is reported with $n_Y = \sqrt{5/3}$ (GUT convention), then at one loop $b_1 = \frac{41}{10}$; with the SM convention ($n_Y = 1$), $b_1 = \frac{41}{6}$.[1] Across heavy thresholds $\mu \simeq m_{\rm th}$ apply matching $g_i(\mu^+) = g_i(\mu^-)\,[1 + \delta_i^{\rm th}]$ with stated $\delta_i^{\rm th}$.

Electroweak angle at M_Z

Define $g_Y := n_Y\, g_1$ so that the electroweak covariant derivative reads $D_\mu \supset i\, g_Y Y B_\mu + i\, g_2 T^i W^i_\mu$. Then

$$\boxed{\sin^2\theta_W(M_Z) \;=\; \frac{g_Y^2(M_Z)}{g_Y^2(M_Z) + g_2^2(M_Z)} \;=\; \frac{n_Y^2\, g_1^2(M_Z)}{n_Y^2\, g_1^2(M_Z) + g_2^2(M_Z)}}. \tag{18.3}$$

Both (g_1, g_2) at M_Z come from (18.1) plus (18.2) with the same n_Y used in building K.

Uncertainties and ellipsoid tests

Let $\mathbf{u}_{\rm ref} = (g_1, g_2, g_3)^\top(\mu_{\rm ref})$ from the anchor, with covariance $\Sigma_{\rm anchor}$ derived from (h_{AB}, E) errors. Propagate to M_Z by linearizing the RG map $\mathbf{u}(M_Z) = \mathcal{R}(\mathbf{u}_{\rm ref}, \vartheta)$ around the best-fit nuisance ϑ (thresholds, top masses, scheme):

$$\Sigma_{M_Z} \;\approx\; J\,\Sigma_{\rm anchor}\,J^\top \;+\; \Sigma_{\rm RG}, \qquad J = \left.\frac{\partial\mathcal{R}}{\partial\mathbf{u}_{\rm ref}}\right|_*,$$

where $\Sigma_{\rm RG}$ collects parametric and truncation uncertainties. Define the *coupling ellipsoid* residual at M_Z relative to the experimental central vector $\widehat{\mathbf{u}}_{\rm exp}$:

$$\chi_g^2 \;=\; \Big(\mathbf{u}(M_Z) - \widehat{\mathbf{u}}_{\rm exp}\Big)^\top \Big(\Sigma_{M_Z} + \Sigma_{\rm exp}\Big)^{-1} \Big(\mathbf{u}(M_Z) - \widehat{\mathbf{u}}_{\rm exp}\Big).$$

Similarly, for the weak angle use the scalar residual

$$\chi_\theta^2 \;=\; \frac{\Big[\sin^2\theta_W(M_Z) - \widehat{s}_W^2\Big]^2}{\sigma_{s_W}^2}, \qquad \sigma_{s_W}^2 \approx \nabla s_W^\top \Sigma_{M_Z}^{(1,2)} \nabla s_W,$$

with $\nabla s_W = \partial(\sin^2\theta_W)/\partial(g_1, g_2)$ taken from (18.3) and $\Sigma_{M_Z}^{(1,2)}$ the 2×2 sub-covariance for (g_1, g_2). *Decision:* accept if χ_g^2 and χ_θ^2 fall inside the declared confidence windows (recorded under `/predictions/ellipsoids`).

[1] More generally $b_1 \propto \sum_{\rm reps} d(R)\, Y^2$; we record the actual b_1 used under `/rg/flows` together with the n_Y choice.

Sensitivity and differential diagnostics

To diagnose geometric origins of tension, evaluate the anchor Jacobian w.r.t. (h_{AB}, E):

$$\frac{\partial g_I^{-2}}{\partial h_{AB}} = \kappa_G\, E^A{}_{(I)} E^B{}_{(I)}, \qquad \frac{\partial g_I^{-2}}{\partial E^A{}_{(J)}} = 2\,\kappa_G\, h_{AB}\, E^B{}_{(I)}\, \delta_{IJ},$$

plus the $K \to (O, D)$ variation if off-diagonal mixing is present. Map these to $\partial \sin^2\theta_W / \partial(h, E)$ using the chain rule and report which Lie–metric directions most affect $\sin^2\theta_W$.

Acceptance (precision).

(i) HDF5 contains the full anchor $(h_{AB}, E, K, O, D, n_Y)$ and the RG pack (beta tables, thresholds, scheme tags); (ii) the ellipsoid tests pass at the declared confidence; (iii) uncertainty budgets separate anchor vs. RG errors.

AR (artifacts).

RG notebooks for 1–2 loop flows; YAML manifest of thresholds/schemes; HDF5 keys:

- `/sm_map/lie_metric`: h_{AB}, units `-`.
- `/sm_map/embeddings`: $E^A{}_{(I)}$.
- `/sm_map/K`, `/sm_map/O`, `/sm_map/D`: Gram & diagonalization.
- `/rg/flows/{mu,beta,g123}`: grids and flows; attrs `scheme`, `order`, `thresholds`.
- `/predictions/thetaW_MZ`: value, error, n_Y.
- `/predictions/ellipsoids/{g_MZ,thetaW}`: centers/covariances and pass/fail flags.

18.2 (S, T, U), EDM floors, NSI textures

Aim.

Turn the spectral action into tight, falsifiable *precision* predictions in three pillars: (i) oblique electroweak parameters (S, T, U); (ii) irreducible electric–dipole *floors*; (iii) neutrino non–standard interaction (NSI) *textures*. Each produces either an ellipsoid/envelope test (two–sided) or a one–sided D kill window.

Oblique parameters (S, T, U)

Let $\Pi_{XY}(q^2)$ denote the transverse vacuum–polarization functions for $X, Y \in \{\gamma, Z, W\}$:

$$i\int d^4x\, e^{iqx}\, \langle J^X_\mu(x) J^Y_\nu(0)\rangle = \Big(g_{\mu\nu} q^2 - q_\mu q_\nu\Big)\, \Pi_{XY}(q^2).$$

We work with the *new-physics shifts* $\Delta\Pi_{XY}(q^2) := \Pi_{XY}(q^2) - \Pi^{\rm SM}_{XY}(q^2)$ and their derivatives at $q^2 = 0$. With $s_W := \sin\theta_W$, $c_W := \cos\theta_W$, and $\alpha = e^2/4\pi$,

$$\alpha\, S = 4 s_W^2 c_W^2 \Big[\Delta\Pi'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{s_W c_W}\,\Delta\Pi'_{Z\gamma}(0) - \Delta\Pi'_{\gamma\gamma}(0)\Big], \tag{18.4}$$

$$\alpha\, T = \frac{\Delta\Pi_{WW}(0)}{M_W^2} - \frac{\Delta\Pi_{ZZ}(0)}{M_Z^2}, \tag{18.5}$$

$$\alpha\, U = 4 s_W^2 \Big[\Delta\Pi'_{WW}(0) - c_W^2 \Delta\Pi'_{ZZ}(0) - 2 s_W c_W\, \Delta\Pi'_{Z\gamma}(0) - s_W^2 \Delta\Pi'_{\gamma\gamma}(0)\Big]. \tag{18.6}$$

Matching. One–loop spectral fields $(\rho, \eta, \chi^\mu, T)$ match onto EW current–current operators; in the SMEFT limit,

$$S \simeq 16\pi v^2\, C_{HWB}, \qquad T \simeq -\frac{v^2}{2\alpha}\, C_{HD}, \qquad U = 0 \text{ at dim-6 (custodial) (+ higher orders).}$$

Here $v = 246\,\text{GeV}$ and the Wilsons inherit geometry via the Lie–metric and overlaps.

Ellipsoid test.

Collect the predicted (S, T, U) with covariance $\Sigma_{\rm th}$ (from parameter and truncation errors). Let $(\widehat{S}, \widehat{T}, \widehat{U})$ and $\Sigma_{\rm exp}$ be the global-fit mean and covariance. The test statistic

$$\chi^2_{STU} = \big(\vec{X} - \widehat{\vec{X}}\big)^\top \big(\Sigma_{\rm th} + \Sigma_{\rm exp}\big)^{-1} \big(\vec{X} - \widehat{\vec{X}}\big), \quad \vec{X} = (S, T, U)^\top,$$

must lie within the declared ellipsoid (e.g. 95%). Record center, axes and pass/fail.

HDF5 keys (oblique).

- `/predictions/oblique/{S,T,U}` (scalars), `/predictions/oblique/cov` (3×3).
- `/predictions/ellipsoids/STU` (center, cov, level), pass flag.
- `/rg/flows/thetaW_at_MZ` for the s_W, c_W used in (18.4)–(18.6).

EDM floors (irreducible, one–sided)

Define dipole and Weinberg operators at the electroweak scale

$$\mathcal{L} \supset -\frac{i}{2} d_f\, \bar{f}\, \sigma^{\mu\nu}\gamma_5 f\, F_{\mu\nu} - \frac{1}{6} w\, f^{abc} G^a_{\mu\nu} G^{b\,\nu}{}_{\rho}\, \tilde{G}^{c\,\rho\mu} + \cdots .$$

Spectral CP–odd invariants (e.g. phases in overlap textures or η–T mixing) generate Wilsons $C_{f\gamma}$, C_W with a *minimal* non–cancellable size. At leading order,

$$d_f(\mu_{\rm had}) = \eta_f\, 2\sqrt{2}\, v\, \Im C_{f\gamma}(\mu_{\rm EW}) \;+\; \xi_f\, w(\mu_{\rm had}) \;+ \cdots ,$$

with RG factors η_f, ξ_f (QCD/QED mixing). Two–loop Barr–Zee topologies yield

$$\frac{d_e}{e} \sim \frac{\alpha}{(4\pi)^3} \frac{m_e}{v^2} \sum_\phi \kappa_\phi \, \Im\!\left[y_e y_\phi^2\right] F\!\left(\frac{m_\phi^2}{m_{\text{loop}}^2}\right),$$

setting an *irreducible floor* $|d_e|_{\text{floor}}$ (and similarly for nucleons via C_W). We declare a one–sided D window:

$$\text{Kill if} \quad |d|_{\text{exp}}^{\text{UL}} < |d|_{\text{floor}} \, (1 - \delta_{\text{had}}),$$

where δ_{had} encodes hadronic/nuclear matrix–element systematics.

HDF5 keys (EDM).

- `/predictions/edm/{de,dn,...}_floor` (values at μ_{had}) + uncertainties.
- `/windows/kill/edm` (one–sided bounds, confidence, δ_{had}).
- `/rg/flows/edm` (RG factors η_f, ξ_f and scales).

NSI textures (matter potential fingerprints)

In matter, oscillations see the Hermitian NSI matrix $\varepsilon_{\alpha\beta}$ via the effective Hamiltonian

$$H = \frac{1}{2E} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger \; + \; \sqrt{2} G_F N_e \begin{pmatrix} 1 + \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix},$$

with $\varepsilon_{\alpha\beta} = \sum_{f=e,u,d} Y_f \, \varepsilon_{\alpha\beta}^f$ and $Y_f = N_f / N_e$. The *spectral texture* predicts (energy–dependent) patterns

$$\varepsilon_{\alpha\beta}(E) \; = \; \sum_r c_r(E) \left[\mathcal{O}_r(\text{fiber overlaps, } \chi^\mu\text{-threads})\right]_{\alpha\beta},$$

often constraining ranks, phases, or sum rules (e.g. $\varepsilon_{\mu\mu} = \varepsilon_{\tau\tau}$ at leading order under custodial/iso-thread symmetry).

Likelihood and orthogonal templates.

We build an oscillation likelihood $\mathcal{L}_{\text{NSI}}$ on appearance/disappearance channels with *orthogonal templates* (constructed to be minimally degenerate with $\theta_{23}, \delta_{\text{CP}}$), and test against an envelope $\mathcal{W}_{\text{NSI}}$:

$$-2 \log \Lambda_{\text{NSI}} = -2 \log \frac{\max_{\text{SM}} \mathcal{L}}{\max_{\text{SM+NSI in } \mathcal{W}_{\text{NSI}}} \mathcal{L}} \quad \overset{\text{calib}}{\sim} \quad \chi_k^2 \text{ (or injection CDF)},$$

with a D *kill* if the best-fit texture exits $\mathcal{W}_{\text{NSI}}$ at the declared confidence.

HDF5 keys (NSI).

- `/predictions/nsi/eps_matrix`(E): datasets for $\varepsilon_{\alpha\beta}(E)$ (axes: E).

- `/windows/envelopes/nsi`: per–entry or norm bounds (e.g. $\|\varepsilon\|_\infty$).
- `/predictions/nsi/templates`: orthogonal templates and their Fisher overlaps.

Acceptance.

1. **STU ellipsoid:** χ^2_{STU} within the declared confidence region; keys and covariances stored.
2. **EDM floor (one–sided):** for each species, $|d|^{\mathrm{UL}}_{\mathrm{exp}} \geq |d|_{\mathrm{floor}}(1-\delta_{\mathrm{had}})$; otherwise *kill*.
3. **NSI envelope:** best-fit texture inside $\mathcal{W}_{\mathrm{NSI}}$ with declared confidence; orthogonal-template pulls below thresholds.

Uncertainty budgets (anchor, RG, hadronic, detector) must be itemized in HDF5 and propagated to the decision statistics (ellipsoids/envelopes/D windows).

Chapter 19

Tabletop/Noise Signatures

19.1 Colored-Zeno filters; opto-mech/atom interferometry

Aim.

Engineer *colored–Zeno* control sequences that sculpt the frequency response seen by system–meter channels, and map the resulting noise fingerprints to laboratory observables in opto–mechanical and atom–interferometric platforms. We provide (i) a convex LMI synthesis for worst–case attenuation under spectral uncertainty, and (ii) device–level transfer functions linking spectral noise to measured spectra, rates, and phase variances.

Control filters and worst–case Zeno synthesis (convex LMIs)

Let $u(t)$ be a real–valued control envelope on $t \in [0, T]$ that modulates the relevant coupling (e.g. toggling frame or dispersive meter strength). Define the filter kernel

$$Y(\omega) \;=\; \int_0^T u(t)\, e^{i\omega t}\, dt, \qquad F(\omega) \;=\; |Y(\omega)|^2.$$

For a stationary colored bath with scalar noise PSD $S(\omega)$ (or matrix S_{ab}), the leading dephasing/heating functional is

$$\Gamma[u] \;=\; \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega)\, F(\omega)\, w(\omega)\, d\omega,$$

where $w(\omega)$ encodes the channel–specific kernel (e.g. $w(\omega) = \omega^{-2}$ for pure dephasing toggling, $w(\omega) = 1$ for force channels).

Discrete SDP formulation.

Sample ω on a grid $\{\omega_j\}_{j=1}^M$, and discretize $u(t)$ on $\{t_n\}_{n=1}^N$ with step Δt. Let $u \in \mathbb{R}^N$ and $b_j \in \mathbb{C}^N$ with $(b_j)_n = \Delta t\, e^{i\omega_j t_n}$. Then

$$F(\omega_j) = |b_j^\top u|^2 \;=\; \mathrm{Tr}\Big(B_j\, U\Big), \quad B_j := b_j b_j^*, \quad U := u\, u^\top \succeq 0,\ \mathrm{rank}(U) = 1.$$

Given an uncertainty envelope $S(\omega) \in \mathcal{S}$ with upper bounds $S(\omega_j) \le \overline{S}_j$, the *robust* colored–Zeno design is

$$\begin{aligned} \text{minimize} \quad & \tau \\ \text{subject to} \quad & \mathrm{Tr}(B_j U) \;\le\; \tau/(\overline{S}_j\, w_j), \qquad j = 1, \dots, M, \\ & \mathrm{Tr}(R\, U) \;\le\; P_{\max}, \\ & U \succeq 0 \quad \text{(rank-1 relaxed)}, \end{aligned} \tag{19.1}$$

where R encodes control power or slew penalties (e.g. finite-difference quadrature of $\int u^2$ and $\int \dot{u}^2$) and $w_j := w(\omega_j)$. Recover u from the dominant eigenvector of U; an eigen–gap $\gg 1$ certifies near–rank–1 solutions. Add linear constraints for hardware: $u_{\min} \le u_n \le u_{\max}$, minimum dwell times, band–limits, etc.

Acceptance (filters).

(i) The synthesized U is $\approx$ rank–1; (ii) measured spectra confirm that $\widehat{F}(\omega)$ obeys the SDP bounds within calibration error; (iii) the observed *Zeno attenuation* factor $\widehat{\Gamma}/\Gamma_{\mathrm{ref}}$ matches prediction over the declared frequency bands.

Opto–mechanics: mapping spectral noise to observables

Consider a linearized cavity–mechanical system with mechanical susceptibility

$$\chi_m(\omega) \;=\; \frac{1}{m\Big(\omega_m^2 - \omega^2 - i\omega\gamma_m\Big)},$$

total force PSD $S_{FF}^{\mathrm{tot}}(\omega) = S_{FF}^{\mathrm{th}} + S_{FF}^{\mathrm{ba}} + S_{FF}^{\mathrm{spec}}$, and amplitude– or phase–modulated probing that implements $u(t)$ and thus $F(\omega)$.

Heating and linewidth.

The *spectral* contribution to the phonon influx is

$$\dot{n}_{\mathrm{spec}} \;=\; \frac{1}{2\pi\hbar\omega_m}\int_{-\infty}^{\infty} S_{FF}^{\mathrm{spec}}(\omega)\, |\chi_m(\omega)|^2\, F(\omega)\, d\omega.$$

The measured linewidth $\gamma_{\mathrm{eff}} = \gamma_m + \Delta\gamma$ receives $\Delta\gamma \propto \partial_\omega \Big[S_{FF}^{\mathrm{spec}}(\omega)|\chi_m|^2 F\Big]_{\omega\approx\omega_m}$, providing a narrow–band *kill test* if the observed $\Delta\gamma$ falls below the predicted irreducible floor for

any admissible $S_{FF}^{\rm spec}$ in the declared DoV.

Output spectrum and SQL relation.

With homodyne angle ζ and transduction $G(\omega)$, the output noise is

$$S_{II}(\omega) \;=\; S_{II}^{\rm imp}(\zeta) + \Big|G(\omega,\zeta)\Big|^2 \,|\chi_m(\omega)|^2\, S_{FF}^{\rm tot}(\omega),$$

and satisfies the imprecision–backaction constraint $S_{II}^{\rm imp}\, S_{FF}^{\rm ba} \geq \hbar^2/4$ (up to inefficiencies). Colored–Zeno shaping modifies G or the effective $S_{FF}^{\rm tot}$ via $F(\omega)$.

Atom interferometry: sequence filters and likelihoods

For a light–pulse sequence with pulses at $\{t_k\}$ and momentum kicks $\{\alpha_k\}$, the *acceleration* filter is

$$H_a(\omega) \;=\; \sum_k \alpha_k\, e^{i\omega t_k}\, \frac{1-e^{i\omega\Delta t_k}}{i\omega}, \qquad F_a(\omega) \;=\; |H_a(\omega)|^2.$$

A $\pi/2$–π–$\pi/2$ Mach–Zehnder has $F_a(\omega) = 16\,\sin^4(\omega T/2)$ (for ideal square pulses), producing notches at $\omega = 2\pi n/T$. The phase variance reads

$$\mathrm{Var}\,\phi \;=\; \frac{1}{2\pi}\int S_{aa}^{\rm tot}(\omega)\, F_a(\omega)\, d\omega,$$

where $S_{aa}^{\rm tot}$ includes the spectral–medium acceleration channel. Colored–Zeno composite pulses (via (19.1)) push F_a's weight out of sensitive bands; the per–run likelihood compares $(\widehat{\phi},\ \widehat{\mathrm{Var}\,\phi})$ to the predicted envelope given $(S_{aa}^{\rm spec},\ \mathrm{DoV})$.

HDF5 keys (tabletop) and decision windows

- `/tabletop/filters`: $\{t_n, u_n\}$, $F(\omega_j)$, SDP certificates (dominant eigenvalue, gap).
- `/tabletop/opto`: $\{\omega_m, \gamma_m, m\}$, $S_{FF}^{\rm spec}(\omega)$, $S_{II}(\omega)$, $\dot{n}_{\rm spec}$, $\Delta\gamma$.
- `/tabletop/ai`: pulse times $\{t_k\}$, kicks $\{\alpha_k\}$, $F_a(\omega)$, $\widehat{\phi}$, $\widehat{\mathrm{Var}\,\phi}$.
- `/windows/kill/tabletop`: bandwise bounds on $S_{FF}^{\rm spec}$ and $S_{aa}^{\rm spec}$, plus one–sided rules (e.g. $\dot{n}_{\rm exp} < \dot{n}_{\rm floor}$ triggers *kill*).

Acceptance.

Device–calibrated $F(\omega)/F_a(\omega)$ match SDP targets within tolerance; measured rates/spectra fall inside the predicted envelopes; one–sided D tests are attached to heating/phase–variance floors for declared bands.

19.2 Noise spectra & device-level mapping

Aim.

Provide a device–agnostic pipeline that maps the *spectral–medium* descriptors (e.g. force, acceleration, phase–diffusion PSDs) into *measured* spectra and time–domain statistics for specific platforms (opto–mechanical cavities, atomic interferometers, waveguides, optional superconducting circuits). The mapping is calibration–driven and auditable: every transfer element and windowing choice is stored and uncertainty–propagated.

Measurement chain and transfer matrices

Let $s(\omega) \in \mathbb{C}^p$ collect the relevant medium channels (*inputs*: force F, acceleration a, phase–diffusion ξ_ϕ, ...), $y(\omega) \in \mathbb{C}^q$ the digitized outputs (photocurrents, phases, displacements), and $n(\omega)$ the instrument noise. The linear response reads

$$y(\omega) \;=\; H(\omega)\, s(\omega) \;+\; n(\omega), \qquad H(\omega) \in \mathbb{C}^{q\times p}. \tag{19.2}$$

Denoting cross–spectra by $S_{xy}(\omega) := \frac{1}{2\pi}\,\mathbb{E}[x(\omega)\, y(\omega)^\dagger]$, we have

$$S_{yy}(\omega) \;=\; H(\omega)\, S_{ss}(\omega)\, H(\omega)^\dagger \;+\; S_{nn}(\omega). \tag{19.3}$$

Colored–Zeno control with filter $F(\omega)$ (or its matrix version F for multichannel sequences) enters as $S_{ss}^{\text{eff}}(\omega) = \mathsf{F}(\omega)\, S_{ss}(\omega)\, \mathsf{F}(\omega)^\dagger$.

State–space view.

For platforms modeled by (A, B, C, D), $H(\omega) = C(i\omega I - A)^{-1}B + D$. We enforce passivity/causality via Kramers–Kronig consistency checks on $\Re H$, $\Im H$ and by pole location $\sigma(A) < 0$ (or marginally stable when appropriate).

Calibration protocol and de–embedding

(C1) **Known injections.** Apply calibrated drives on each input channel (using radiation–pressure, shaker, coil, or phase actuator) to obtain S_{yy}^{inj} and cross–spectra S_{yu}^{inj} with the drive record u.

(C2) **Transfer recovery.** Solve for H by complex least–squares on each frequency bin: $\min_H \|Y - HU\|_F^2$ with Tikhonov ridge $\lambda\|H\|_F^2$ and optional structure (H diagonal/triangular). Propagate covariance to get $\Sigma_H(\omega)$.

(C3) **Zeno de–embedding.** If a control $u(t)$ was active, divide by the known filter: $H \leftarrow H\,\mathsf{F}^{-1}$ where admissible (or fold F into S_{ss}).

(C4) **Noise floor.** With inputs quieted, measure S_{nn} (window–matched). Store both the raw and a whitened estimate (prewhitening FIR W s.t. $WS_{nn}W^\dagger \approx I$).

PSD estimation (Welch/MT).

Estimate S_{yy} by Welch with window w, segment length N_{seg}, overlap ϱ, and (optionally) multitaper; the *equivalent degrees of freedom* ν yield $100(1-\alpha)\%$ confidence belts via χ^2_ν:

$$S^{\text{lo}}_{yy}(\omega) = \frac{\nu\,\widehat{S}_{yy}(\omega)}{\chi^2_{1-\alpha/2,\nu}}, \quad S^{\text{hi}}_{yy}(\omega) = \frac{\nu\,\widehat{S}_{yy}(\omega)}{\chi^2_{\alpha/2,\nu}}.$$

Record $\{w, N_{\text{seg}}, \varrho, \nu\}$ alongside $\widehat{S}$.

Inverse mapping and identification of S_{ss}

Given $\widehat{H}, \widehat{S}_{yy}, \widehat{S}_{nn}$, recover S_{ss} by *PSD–level* constrained least–squares:

$$\begin{aligned} &\min_{S_{ss}(\omega)\succeq 0} \left\| \widehat{S}_{yy}(\omega) - \widehat{H}(\omega)\, S_{ss}(\omega)\, \widehat{H}(\omega)^\dagger - \widehat{S}_{nn}(\omega) \right\|_F^2 \\ &\quad \text{s.t. } S_{ss}(\omega) \text{ Toeplitz–PSD (Bochner)}, \quad S_{ss} \in \mathcal{S}_{\text{DoV}}, \end{aligned} \tag{19.4}$$

where $\mathcal{S}_{\text{DoV}}$ encodes prior structure (e.g. bandedness, Lorentzian sums, power–law envelopes). For single–input p=1, this reduces to the scalar $\widehat{S}_{ss} = (\widehat{S}_{yy} - \widehat{S}_{nn})/|\widehat{H}|^2$ with uncertainty from linear error propagation using Σ_H and Welch's ν.

Low–frequency drifts.

Use Allan variance $\sigma^2_y(\tau)$ and Hadamard variance for drift–dominated bands; link to PSD by the known τ–domain kernels and store both $\widehat{S}$ and $\widehat{\sigma}(\tau)$ to guard against leakage/bias.

Device exemplars (closed formulas)

Opto–mechanics.

With mechanical susceptibility $\chi_m(\omega)$ and transduction $G(\omega)$,

$$S_{II}(\omega) = S^{\text{imp}}_{II} + |G(\omega)|^2\, |\chi_m(\omega)|^2 \left(S^{\text{th}}_{FF} + S^{\text{ba}}_{FF} + S^{\text{spec}}_{FF} \right),$$

where S^{spec}_{FF} is the spectral–medium force PSD shaped by $F(\omega)$.

Atom interferometry.

For acceleration channel with filter $F_a(\omega)$, $\operatorname{Var}\phi = \frac{1}{2\pi}\int S^{\text{tot}}_{aa}(\omega)\, F_a(\omega)\, d\omega$; phase noise spectra follow by linearization about the readout.

Waveguides / photonics.

Phase/delay fluctuations from refractive–index perturbations give $S_{\phi\phi}(\omega) = k_0^2\, L^2\, S_{nn}^{(n)}(\omega)$; connect medium's index channel to $S_{nn}^{(n)}$ via the platform–specific $H(\omega)$.

Decision rules and kill windows (tabletop)

Define per–band bounds $\mathcal{W}_k = [L_k, U_k]$ for medium PSD components after inversion (19.4). With estimated $\widehat{S}_{ss}(\omega)$ and uncertainty belts, declare:

$$\textit{kill} \iff \exists k: \ \widehat{S}_{ss}(\omega) \text{ enters } \mathbb{R} \setminus \mathcal{W}_k \text{ over a set of nonzero measure,}$$

using a multiple–testing control (e.g. Bonferroni or FDR) across frequency bins. For heating floors or phase–variance floors, use one–sided tests against the irreducible bound predicted by the model under the declared DoV.

HDF5 keys (noise/mapping)

- `/tabletop/transfer`: $\widehat{H}(\omega)$, $\Sigma_H(\omega)$, K–K consistency flags.
- `/tabletop/noise/raw`: $\widehat{S}_{yy}(\omega)$, cross–spectra, Welch/MT params $\{w, N_{\text{seg}}, \varrho, \nu\}$.
- `/tabletop/noise/floor`: $\widehat{S}_{nn}(\omega)$, whitening filter W.
- `/tabletop/noise/inverted`: $\widehat{S}_{ss}(\omega)$ with uncertainty belts, constraints used.
- `/windows/kill/tabletop`: band definitions, bound types (two– or one–sided), confidence levels.

Acceptance.

Device–calibrated predictions + kill/decision rules.

Part VII

Inference, Forecasts, and Decision Windows

Chapter 20

Likelihoods and Priors

20.1 CMB/GW/EDM/NSI/Tabletop likelihoods

Purpose.

We present a unified, audit-ready likelihood stack for the principal data classes entering the USFT program: CMB anisotropies, gravitational-wave (GW) propagation/dispersion, precision EDM measurements, neutrino oscillations with possible NSI, and tabletop noise spectroscopy. All likelihoods are written with explicit nuisance/calibration parameters and are accompanied by machine-checkable HDF5 entries. Priors for the spectral parameters are specified in §20.2.

Global structure and notation

Let the parameter vector split as

$$\theta \equiv (\vartheta, \nu), \qquad \vartheta \text{ (physics: MSP, geometry, RG)}, \quad \nu \text{ (nuisance: calibration, beams, noise, backgrounds}$$

Given independent data blocks $\{D_k\}$ with possible cross-covariances, the joint likelihood reads

$$\mathcal{L}(D \mid \theta) \;=\; \left[\prod_k \mathcal{L}_k(D_k \mid \theta)\right] \exp\Big(-\Phi_{\text{cross}}(\{D_k\}, \theta)\Big)\, \mathbf{1}_{\text{gates}}(\theta), \tag{20.1}$$

where Φ_{cross} penalizes modeled inter-dataset correlations if present (e.g. shared calibration & foregrounds), and $\mathbf{1}_{\text{gates}}$ is a hard/soft indicator implementing declared DoV and gate constraints (S/G/TQ/D). For a zero-mean complex Gaussian vector $x \sim \mathcal{N}_{\mathbb{C}}(0, C)$ we use

$$-2 \ln \mathcal{L}_{\text{G}}(x; C) = \ln \det(\pi C) + x^{\dagger} C^{-1} x. \tag{20.2}$$

For stationary time series with one-sided PSD $S(\omega)$ we adopt the Whittle form on tapered segments:

$$-2\ln\mathcal{L}_{\mathrm{W}}(\tilde{d};\,S) \;=\; \sum_{\omega\in\mathcal{B}}\Big(\ln S(\omega)+\frac{|\tilde{d}(\omega)|^2}{S(\omega)}\Big)+\text{const.} \tag{20.3}$$

Robust alternatives (Student-t) are available via a scale-mixture augmentation.

20.1.1 CMB temperature/polarization bandpowers

Let $\widehat{\mathbf{C}}_b$ denote estimated bandpowers for $X,Y\in\{T,E,B\}$ over bands b with covariance $\Sigma^{\mathrm{cmb}}_{bb'}$. The model prediction $\mathbf{C}_b(\theta)$ includes all beam/calibration nuisances:

$$\mathbf{C}_b(\theta)=\mathsf{B}_b(\nu)\,\mathbf{C}^{\mathrm{th}}_b(\vartheta)\,\mathsf{B}_b^{\top}(\nu)\;+\;\mathbf{N}_b(\nu),$$

where B_b encodes beam window and overall gain, and $\mathbf{N}_b$ the noise/foreground residual template. A Gaussian bandpower approximation gives

$$-2\ln\mathcal{L}_{\mathrm{CMB}}=\sum_{b,b'}\Big(\widehat{\mathbf{C}}_b-\mathbf{C}_b(\theta)\Big)^{\top}\Big[\Sigma^{\mathrm{cmb}}\Big]^{-1}_{bb'}\Big(\widehat{\mathbf{C}}_{b'}-\mathbf{C}_{b'}(\theta)\Big). \tag{20.4}$$

For near full-sky, a Wishart form per ℓ may be used; for low SNR bands, a lognormal (offset) mapping of $\mathbf{C}_b$ stabilizes variances. HDF5: `/likelihood/cmb/{bandpowers,cov,beams,calib}`.

20.1.2 Gravitational waves: propagation, $c_T(f,z)$, dispersion

Let $\tilde{d}_k(f)$ be the strain in detector k with response $R_k(f,\nu)$, and $\tilde{h}(f;\vartheta)$ the source template (or stochastic GW background). Propagation through the spectral medium induces a transfer $\mathsf{T}(f,z;\vartheta)$ (phase velocity c_T, attenuation, dispersion). The predicted frequency-domain mean is $\tilde{m}_k=R_k\,\mathsf{T}\,\tilde{h}$ and the noise PSD is $S_{n,k}(f;\nu)$. The Whittle likelihood per detector is

$$-2\ln\mathcal{L}_{\mathrm{GW}}=\sum_k\sum_{f\in\mathcal{B}}\Big[\ln S_{n,k}(f)+\frac{|\tilde{d}_k(f)-\tilde{m}_k(f;\theta)|^2}{S_{n,k}(f)}\Big]. \tag{20.5}$$

For stochastic backgrounds, use cross-correlation of detector pairs $(k\ell)$ with model cross-PSD $S^{\mathrm{GW}}_{k\ell}(f;\theta)$ and the usual Gaussian approximation for the estimator. Selection effects: for event catalogs with detection efficiency $\epsilon(\vartheta)$, include the inhomogeneous Poisson factor $\exp\Big[-\mu(\vartheta)\Big]\prod_i\lambda(d_i\mid\vartheta)$ with μ the expected counts (efficiency-folded). HDF5: `/likelihood/gw/{psd,cross,resp,efficiency}`.

20.1.3 Electric dipole moments (EDM)

Let $\widehat{\mathbf{d}}$ collect EDM measurements (electron, neutron, atoms) with covariance Σ^{edm} and systematic floor parameters ν_{sys}. The prediction $\mathbf{d}(\vartheta)$ is model-dependent (Barr–Zee-type floors, new phases, etc.). A Gaussian likelihood suffices for symmetric errors:

$$-2\ln\mathcal{L}_{\mathrm{EDM}} = \Big(\widehat{\mathbf{d}} - \mathbf{d}(\theta)\Big)^{\top}\Big[\Sigma^{\mathrm{edm}}\Big]^{-1}\Big(\widehat{\mathbf{d}} - \mathbf{d}(\theta)\Big). \tag{20.6}$$

For null bounds, adopt a *one-sided* folded-Gaussian or survival-function penalty $\mathcal{L} \propto \prod_i \frac{1}{2}\,\mathrm{erfc}\Big(\frac{d_i^{\mathrm{floor}}(\vartheta)-d_i^{\mathrm{UL}}}{\sqrt{2}\,\sigma_i}\Big)$. HDF5: `/likelihood/edm/{values,cov,limits}`.

20.1.4 Neutrino oscillations with NSI; near–far

Bin events by baseline/energy (L_j, E_b) and channels $\alpha\to\beta$. Counts $\widehat{n}_{jb}$ are modeled as Poisson with means

$$\lambda_{jb}(\theta) = \Phi_j(\nu)\,\sigma_b(\nu)\,\epsilon_{jb}(\nu)\,P_{\alpha\beta}(L_j, E_b;\vartheta)\,T_{jb},$$

where Φ flux, σ cross-section, ϵ efficiency, T exposure, and $P_{\alpha\beta}$ includes NSI textures and possible CPT-odd terms from the spectral medium. The likelihood is

$$\ln\mathcal{L}_{\mathrm{NSI}} = \sum_{j,b}\Big(\widehat{n}_{jb}\ln\lambda_{jb} - \lambda_{jb} - \ln(\widehat{n}_{jb}!)\Big) \;-\; \frac{1}{2}(\nu-\bar{\nu})^{\top}\Sigma_{\nu}^{-1}(\nu-\bar{\nu}), \tag{20.7}$$

with Gaussian pulls for systematics ν. A joint *near–far* statistic penalizes inconsistencies: $\chi^2_{NF} = (\widehat{\mathbf{n}}^F - \mathsf{M}\,\widehat{\mathbf{n}}^N)^{\top}\Sigma_{NF}^{-1}(\widehat{\mathbf{n}}^F - \mathsf{M}\,\widehat{\mathbf{n}}^N)$, where M is a transfer map (acceptance in §??). HDF5: `/likelihood/nsi/{counts,expo,pulls,NF_map}`.

20.1.5 Tabletop: PSDs, cross-spectra, phase variance

With calibrated transfer $H(\omega;\nu)$ (§19.2), and medium PSD $S_{ss}(\omega;\vartheta)$ shaped by control filters, the predicted output PSD is $S_{yy} = HS_{ss}H^{\dagger} + S_{nn}$. Using Whittle's approximation on Welch/MT estimates:

$$-2\ln\mathcal{L}_{\mathrm{TT}}^{\mathrm{PSD}} = \sum_{\omega\in\mathcal{B}}\Big[\ln\det S_{yy}(\omega) + \mathrm{Tr}\,\Big(\widehat{S}_{yy}(\omega)\,S_{yy}^{-1}(\omega)\Big)\Big]. \tag{20.8}$$

For atom interferometers, add a Gaussian likelihood for the phase-variance observable $\widehat{\mathrm{Var}\,\phi}$ with model $\mathrm{Var}\,\phi(\theta) = \frac{1}{2\pi}\int S_{aa}^{\mathrm{tot}}(\omega)\,F_a(\omega)\,d\omega$. HDF5: `/likelihood/tabletop/{Syy,Snn,H,phasevar}`.

Cross-covariances and gates

If a nuisance (e.g. calibration) is shared across datasets, include blocks in Φ_{cross} of (20.1). Gate consistency enters as hard constraints or as soft penalties; e.g.

$$\Phi_{\text{gates}}(\theta) = \lambda_S \max(0, -\mu_{\min}(\theta))^2 + \lambda_G \max\Big(0, \max(v_{\text{front}}/\text{cEff}) - 1\Big)^2 + \cdots ,$$

with audit traces recorded under `/audits`.

HDF5 keys (likelihood layer)

- `/likelihood/cmb`: `bandpowers`, `cov`, `beam`, `calib`.
- `/likelihood/gw`: `psd`, `cross`, `resp`, `efficiency`.
- `/likelihood/edm`: `values`, `cov`, `limits`.
- `/likelihood/nsi`: `counts`, `expo`, `pulls`, `NF_map`.
- `/likelihood/tabletop`: `Syy`, `Snn`, `H`, `phasevar`.
- `/likelihood/meta`: taper/window params, ν-priors, segmenting, dof.

Acceptance.

Each likelihood component ships with (i) explicit nuisance priors; (ii) calibration artifacts (responses/PSDs) and uncertainty propagation; (iii) gate-compliant flags. The combined posterior (with priors from §20.2) is reproducible from the HDF5 bundle via the documented API.

20.2 Spectral-action parameter priors

Purpose.

We specify a gate-compatible prior layer for the spectral–action parameters that (i) respects positivity/compactness constraints, (ii) carries explicit units, and (iii) integrates with the likelihoods in §20.1 via an auditable YAML/HDF5 schema. The parameter vector is partitioned as

$$\vartheta \equiv \Big(\underbrace{\kappa_G, h_{AB}, E^A{}_{(I)}}_{\text{geometry/YM anchor}} , \quad \underbrace{\theta_{\text{pot}}}_{\text{potential/masses}} , \underbrace{\theta_{\text{GKSL}}}_{\text{open system}} , \quad \underbrace{\theta_{\text{cos}}}_{\text{background/cosmology}} \Big),$$

with support restricted to the declared DoV and the S/G/TQ/Dgates.

Support, constraints, and gates

We enforce hard support through indicator factors:

$$\pi(\vartheta) \;\propto\; \pi_0(\vartheta)\, \mathbf{1}_{\mathrm{DoV}}(\vartheta)\, \mathbf{1}_{\mathrm{gateS}}(\vartheta)\, \mathbf{1}_{\mathrm{gateG}}(\vartheta)\, \mathbf{1}_{\mathrm{gateTQ}}(\vartheta)\, \mathbf{1}_{\mathrm{gateD}}(\vartheta),$$

where, e.g., $\mathbf{1}_{\mathrm{gateTQ}}(\vartheta) = 1$ iff the Choi spectrum of the calibrated channel is nonnegative within a margin (cf. §12.2).

Default priors (dimensionally explicit)

Below, $[x]$ denotes the physical unit of x. We use weakly informative, log-convex choices for positive scales and rotation-invariant choices on Stiefel/orthogonal manifolds.

P1. Geometry / Lie–metric (h_{AB}) and YM anchor.

$$h \sim \mathrm{LKJ}(\eta_h) \text{ on correlations}, \qquad \sigma_h \sim \mathrm{HalfCauchy}(0,\, \tau_h) \text{ on scales}, \qquad [h_{AB}] = 1.$$

Embedding columns $E^A{}_{(I)}$ are given a Stiefel prior $E \in \mathrm{St}(d,k)$ with density $\propto \exp(\mathrm{tr}(Q^\top E))$ where Q can encode mild alignment to a baseline; set $Q = 0$ for noninformative.

P2. Spectral normalization κ_G.

$$\log \kappa_G \sim \mathcal{N}(\mu_\kappa, \sigma_\kappa^2), \qquad [\kappa_G] = [\mathrm{action}]^{-1}.$$

P3. Potential/mass scales θ_{pot}. For each positive mass/energy scale m_i and quartic coupling λ_a:

$$\log m_i \sim \mathcal{N}(\mu_{m_i}, \sigma_{m_i}^2), \qquad \lambda_a \sim \mathrm{HalfNormal}(\sigma_{\lambda_a}), \qquad [m_i] = \mathrm{energy},\ [\lambda_a] = 1.$$

Mixing angles α on compact ranges use $\alpha \sim \mathrm{Uniform}[0,\pi)$ and phases $\varphi \sim \mathrm{Uniform}[0,2\pi)$.

P4. Open-system/GKSL block θ_{GKSL}. Lindblad rates $\gamma_\ell > 0$ and colored-kernel parameters β:

$$\log \gamma_\ell \sim \mathcal{N}(\mu_{\gamma_\ell}, \sigma_{\gamma_\ell}^2), \qquad \beta \sim \mathcal{N}(\bar{\beta}, \Sigma_\beta), \qquad [\gamma_\ell] = \mathrm{time}^{-1}.$$

Positivity/CPTP is enforced either by (i) parametrizing the Choi matrix $J = LL^\dagger$ with a free lower-triangular L, or (ii) adding a soft barrier $\propto \exp(-\lambda_{TQ} \sum_i \max(0, -\lambda_i(J))^2)$.

P5. Cosmology/background block θ_{cos}.

$$H_0 \sim \mathcal{N}(\mu_{H_0}, \sigma_{H_0}^2), \quad \Omega_b, \Omega_c \sim \mathrm{Dirichlet}(\alpha_b, \alpha_c), \quad w_{\mathrm{eff}}(a) \text{ via GP prior}$$

with a Matérn kernel $k(a, a') = \sigma_w^2 \, \mathcal{M}_\nu(|a - a'|/\ell_w)$ and hyperpriors $\log \sigma_w, \log \ell_w \sim \mathcal{N}$; units carried in the GP scale.

Reparameterizations and Jacobians (sampling on $\mathbb{R}^n$)

We work with an unconstrained vector z and transforms $\vartheta = T(z)$:

$$x > 0: \quad x = \mathrm{softplus}(z) = \log(1 + e^z), \quad \log\left|\frac{dx}{dz}\right| = -\log(1 + e^{-z}),$$
$$x \in (a, b): \quad x = a + (b - a)\,\mathrm{logit}^{-1}(z), \quad \log\left|\frac{dx}{dz}\right| = \log(b - a) - 2\log\left(1 + e^{-z}\right).$$

The log-prior includes the Jacobian term, ensuring invariance of the target density.

Gate-compatible soft penalties

Where hard indicators are impractical for gradient-based samplers, we add

$$\Phi_{\mathrm{soft}}(\vartheta) = \lambda_S \, \max(0, -\mu_{\min}(\vartheta))^2 + \lambda_G \, \max(0, \max(v_{\mathrm{front}}/\mathrm{cEff}) - 1)^2 + \lambda_{TQ} \sum_i \max(0, -\lambda_i(J))^2,$$

and sample w.r.t. $\pi_{\mathrm{soft}}(\vartheta) \propto \pi_0(\vartheta)\, e^{-\Phi_{\mathrm{soft}}}$.

YAML schema (auditable priors)

Minimal, machine-read YAML defining each prior with units and transforms:

```yaml
# /priors/global.yaml
version: v7
units_system: "SI"  # or "natural(c=hbar=1)"
blocks:
  kappa_G:
    dist: lognormal
    params: {mu: -2.0, sigma: 1.0}
    units: "[action]^-1"
    transform: identity
  lie_metric:
    dist: lkj
    params: {eta: 1.0}
    scale: {dist: halfcauchy, params: {scale: 1.0}}
    manifold: SPD
    units: "1"
  embeddings:
```

```
    dist: stiefel
    params: {d: 12, k: 3, bias_matrix: null}
    units: "1"
  masses:
    m_phi:
      dist: lognormal
      params: {mu: 0.0, sigma: 2.0}
      units: "GeV"
    m_eta:
      dist: lognormal
      params: {mu: 2.0, sigma: 1.0}
      units: "GeV"
  gksl:
    gamma:
      dist: lognormal
      params: {mu: -3.0, sigma: 1.0}
      units: "1/s"
    choi_param:
      dist: cholesky  # J = L L^†
      size: [n, n]
      units: "1"
  cosmology:
    H0:
      dist: normal
      params: {mu: 67.4, sigma: 1.0}
      units: "km s^-1 Mpc^-1"
    weff_gp:
      kernel: matern32
      params: {sigma: 0.1, ell: 0.2}
      units: "1"
gates:
  hard: {DoV: true}
  soft: {lambda_S: 1e4, lambda_G: 1e4, lambda_TQ: 1e4}
```

HDF5 storage

Store the YAML text under `/priors/global.yaml` (as a UTF-8 string dataset) and a normalized, columnar expansion under `/priors/table` with fields: `name` (string), `dist` (string), `params` (JSON string), `units` (string), `transform` (string). Attach `hash_sha256`

and `schema_version` attributes for provenance.

Synthetic-data generators and unit tests

For each likelihood block, ship a generator that draws $\vartheta \sim \pi(\cdot)$ and produces D via the forward model, writing both to HDF5. Unit tests must verify: (i) unit consistency of prior hyperparameters; (ii) support compliance (DoV, gates); (iii) reproducibility given a fixed RNG seed.

AR.

YAML schema as in §20.2; unit tests for transforms/Jacobians; synthetic-data generators per block (CMB/GW/EDM/NSI/Tabletop), with manifests stored under `/runs` and prior hashes recorded in `/audits`.

Chapter 21

Forecasts, Sensitivity, and Model Comparison

21.1 Fisher analyses; NUTS/MCMC; Bayes factors

Scope.

This chapter formalizes three complementary layers: (i) *Fisher forecasts* for fast sensitivity and design; (ii) *full posterior sampling* (NUTS/MCMC) with gate-aware priors from §20.2; (iii) *model comparison* via evidences/Bayes factors. All results are restricted to the declared DoV and audited against S/G/TQ/D.

Fisher information and forecasted constraints

Let $\vartheta \in \mathbb{R}^{n_\theta}$ be the parameter vector and d the stacked data vector across experiments with mean $\mu(\vartheta)$ and covariance $C(\vartheta)$. Around a reference point $\vartheta_\star$ in the DoV, the Gaussian-loglikelihood expansion yields the Fisher matrix

$$F_{ij} \;=\; \Big(\partial_i\mu\Big)^{\top} C^{-1}\Big(\partial_j\mu\Big) \;+\; \frac{1}{2}\,\mathrm{Tr}\Big[\,C^{-1}(\partial_i C)\,C^{-1}(\partial_j C)\,\Big]\;\Bigg|_{\vartheta=\vartheta_\star}, \tag{21.1}$$

with $\partial_i \equiv \partial/\partial\vartheta_i$. For block-diagonal $C = \mathrm{diag}(C^{(b)})$ (independent experiments) we have $F = \sum_b F^{(b)}$. Forecasted (Cramér–Rao) marginalized 1σ errors follow from $\Sigma \equiv F^{-1}$ as $\sigma(\vartheta_i) = \sqrt{\Sigma_{ii}}$.

Profiling nuisance parameters.

Partition $\vartheta = (\theta, \nu)$ (parameters of interest vs. nuisances). With the Schur complement,

$$F_{\mathrm{marg}}(\theta) \;=\; F_{\theta\theta} \;-\; F_{\theta\nu}\,F_{\nu\nu}^{-1}\,F_{\nu\theta}, \tag{21.2}$$

giving forecasted constraints on θ after profiling ν.

Design and SNR links.

For linear models with C independent of ϑ, the expected detection SNR for a template $t(\vartheta)$ is $\mathrm{SNR}^2(\vartheta) = t^\top C^{-1} t$ and the local curvature along direction u is $u^\top F u$. We use these to allocate observing time/bandpasses to maximize either $\det F$ (D-optimal) or $\mathrm{Tr}(F)$ (A-optimal).

Gate-aware Fisher.

Gradients $\partial_i \mu$ are computed only at configurations that pass the gates; if a step crosses a boundary trigger (e.g. $v_{\text{front}} > \text{cEff}$), the derivative is replaced by a one-sided finite difference within the DoV. Forecast ellipses violating gates are discarded.

Artifacts.

Store F, Σ, and gradient slices under `/inference/fisher/{F,Sigma,grads}` with attributes: experiment tags, units, and $\vartheta_\star$.

Full posterior sampling: NUTS/MCMC

We target the gate-compatible posterior

$$\log \mathcal{P}(\vartheta \mid D) = \log \mathcal{L}(D \mid \vartheta) + \log \pi(\vartheta) - \Phi_{\text{soft}}(\vartheta) \quad \text{on the support of } \mathbf{1}_{\text{DoV}} \times \mathbf{1}_{\text{gates}},$$

where π and Φ_{soft} are defined in §20.2.

NUTS configuration (defaults).

Dual-averaging step-size adaptation (warmup = 1000), diagonal then dense mass-matrix adaptation, target acceptance 0.8–0.9. We run ≥ 4 independent chains.

Diagnostics and acceptance.

Let $\hat{R}$ denote rank-normalized split-$\hat{R}$ and $\mathrm{ESS}_{\text{bulk}}, \mathrm{ESS}_{\text{tail}}$ the effective sample sizes.

- Convergence: $\hat{R} < 1.01$ for all scalar summaries; no persistent divergences.
- Efficiency: $\mathrm{ESS}_{\text{bulk}}, \mathrm{ESS}_{\text{tail}} \geq 400$ per parameter block.
- Energy behavior: BFMI > 0.3; no pathological energy–step correlations.
- Gate compliance: samples failing gates are either rejected (hard) or heavily penalized (soft).

Posterior predictive checks (PPC) are performed for each observable family.

Bridging from Fisher.

Initialize chains at $\vartheta_\star$ with Gaussian proposal $\mathcal{N}(\vartheta_\star, \Sigma)$ from F^{-1}; optionally anneal likelihood temperature from $\tau = 0.2 \to 1$.

Artifacts.

Write samples, warmup diagnostics, and divergences to HDF5: `/inference/mcmc/{chains, diag, warmup}`; include `log_post`, `log_like`, `log_prior` fields and chain seeds.

Model comparison and Bayes factors

Let $\mathcal{M}_0, \mathcal{M}_1$ be competing models (e.g. with/without a spectral interaction). The Bayes factor is $B_{10} = \mathcal{Z}_1/\mathcal{Z}_0$, with evidence $\mathcal{Z} = \int \mathcal{L}(D \mid \vartheta)\, \pi(\vartheta)\, d\vartheta$.

Evidence estimators (audited).

(a) *Laplace (Fisher) approximation* near MAP $\hat{\vartheta}$: $\log \mathcal{Z} \approx \log \mathcal{L}(\hat{\vartheta}) + \log \pi(\hat{\vartheta}) + \frac{1}{2} \log |(2\pi)^n \hat{\Sigma}|$, where $\hat{\Sigma}^{-1} = -\nabla^2 \log \mathcal{P}$.

(b) *Thermodynamic integration* over inverse temperature $\beta \in [0, 1]$: $\log \mathcal{Z} = \int_0^1 \mathbb{E}_\beta[\log \mathcal{L}]\, d\beta$ with $\mathbb{E}_\beta$ taken under $\propto \mathcal{L}^\beta \pi$ (annealed NUTS).

(c) *Bridge sampling / SMC* for cross-checking difficult posteriors.

For nested models ($\mathcal{M}_0$ is $\mathcal{M}_1$ with $\phi = \phi_0$), the Savage–Dickey density ratio applies: $B_{10} = \pi(\phi = \phi_0 \mid \mathcal{M}_1)/p(\phi = \phi_0 \mid D, \mathcal{M}_1)$, provided regularity and separable priors.

Occam penalties and gates.

Evidence naturally penalizes superfluous volume; gate-restricted priors reduce effective prior mass to the admissible DoV, making comparisons sharper and auditable.

Kill-window translation.

Posterior mass in the predeclared kill region $\mathcal{W}_{\text{kill}}$ triggers D actions: if $p(\vartheta \in \mathcal{W}_{\text{kill}} \mid D) \geq \tau_{\text{kill}}$ (e.g. 0.95), the hypothesis is rejected; for model-level statements use thresholds on $\log B_{10}$.

Artifacts.

`/inference/evidence/{laplace, thermoint, bridge}` with grids, annealing schedules, and numerical uncertainties; decision tables under `/windows/kill` are updated with posterior probabilities and Bayes factors.

Putting it together: pipeline summary

1. **Linearize & Fisher** at a gate-passing $\vartheta_\star$; store F, Σ.
2. **Warm-start NUTS** with Σ; run chains with gate-aware priors.
3. **Diagnostics & PPC** ($\hat{R}$, ESS, BFMI, divergences); fail if thresholds unmet.
4. **Evidence** via Laplace and thermodynamic integration; cross-check by bridge sampling.
5. **Decisions**: report forecast vs. posterior intervals; map to D kill windows and record Bayes factors for competing hypotheses.

Acceptance (Inference layer).

A release passes if: (i) Fisher & posterior summaries agree to first order in the high-SNR limit; (ii) NUTS diagnostics meet thresholds; (iii) evidences are stable to method choice within stated numerical errors; (iv) all artifacts are written with units and hashes.

21.2 Per-observable kill windows & stopping rules

Aim.

This section operationalizes D: for each observable O we define *kill/decision windows*, map posterior and frequentist evidence to binary actions (proceed/stop/revise), and specify sequential stopping with error control. All rules act *within* the declared DoV.

Definitions: windows and indicators

Let $\widehat{O}$ denote the measured summary (or vector) for an observable family and let $\mathcal{W}_O = [L_O, U_O]$ be its admissible window (units and confidence/credibility level are attached as attributes). Define the indicator

$$\mathbf{1}_{\text{pass}}(O; D) = \begin{cases} 1, & \widehat{O} \in \mathcal{W}_O, \\ 0, & \widehat{O} \notin \mathcal{W}_O. \end{cases}$$

Windows can be *two-sided* (interval) or *one-sided* ($(-\infty, U_O]$ or $[L_O, \infty)$), with optional tightening factors for device/systematics slacks. Windows and metadata are stored under `/windows/kill` (cf. V7 schema).

Bayesian (posterior) decision rule

Let ϑ be the model parameters; let $\mathcal{W}^O_{\text{kill}}(\vartheta)$ denote the parameter region that implies an *effective* violation for O (e.g. predicted $O(\vartheta)$ outside $\mathcal{W}_O$ once systematics are marginalized). The posterior kill probability is

$$p^O_{\text{kill}} \;=\; \Pr\Big(\vartheta \in \mathcal{W}^O_{\text{kill}} \mid D\Big) \;=\; \int_{\mathcal{W}^O_{\text{kill}}} \mathcal{P}(\vartheta \mid D)\, d\vartheta. \tag{21.3}$$

Rule (Gate D-Bayes). Choose a confidence level $\tau_{\text{kill}} \in [0.9, 0.99]$. Then

$$p^O_{\text{kill}} \;\geq\; \tau_{\text{kill}} \quad\Longrightarrow\quad \textsc{Kill}(O), \qquad p^O_{\text{kill}} \;\leq\; \tau_{\text{safe}}\ (< \tau_{\text{kill}}) \quad\Longrightarrow\quad \textsc{Proceed}(O),$$

else Hold/Collect (insufficient evidence). Record $\{p^O_{\text{kill}}, \tau_{\text{kill}}, \tau_{\text{safe}}\}$.

Frequentist decision rule

Let $T_O(D)$ be a test statistic with known (or resampled) null distribution under the model-hypothesis H_0 restricted to DoV. For two-sided windows, define

$$p_O \;=\; \Pr_{H_0}\Big(|T_O| \geq |T_O^{\text{obs}}|\Big), \qquad \alpha_O \in (0,1) \text{ (per-observable level).}$$

Rule (Gate D-Freq). If $p_O \leq \alpha_O$ and the effect has the *kill direction* (outside $\mathcal{W}_O$), then KILL(O); if $p_O > \alpha_O$ and $\widehat{O} \in \mathcal{W}_O$, then PROCEED(O); otherwise HOLD/COLLECT. For multiple observables use a predeclared correction (e.g. Holm–Bonferroni over a family or per-portfolio false-discovery rate q).

Sequential stopping with spending functions

Suppose data arrive in K stages ($k = 1, \ldots, K$). Define cumulative log-evidence $Z_k \equiv \log B_{10}^{(1:k)}$ or a cumulative z-score Z_k^{freq}. Choose monotone boundaries $a_k < b_k$ (efficacy/futility) via:

- **Bayesian SPRT.** Stop for efficacy if $Z_k \geq b_k$, for futility if $Z_k \leq a_k$, else continue. Typical constants: $b_k = \log(10)$, $a_k = -\log(10)$, or use O'Brien–Fleming–like boundaries increasing with k.

- **α-spending (frequentist).** With target family-wise α_{tot}, choose a spending function $\mathcal{A}(t)$, $t = k/K$, and stagewise critical values so that $\sum_k \alpha_k = \alpha_{\text{tot}}$. Stop when $p_{O,k} \leq \alpha_k$ (efficacy) or when conditional power falls below a futility threshold.

Gate. Any early stop that implies violation of $\mathcal{W}_O$ triggers KILL(O) and emits a revision ticket (see governance chapter).

Combining observables and orthogonal templates

When several observables probe the same hypothesis H with partially correlated uncertainties, construct a joint statistic

$$\chi^2_H \;=\; (\widehat{\mathbf{O}} - \mathbf{O}_H)^\top \mathbf{C}^{-1} (\widehat{\mathbf{O}} - \mathbf{O}_H),$$

and test against an *orthogonal* template bank to break degeneracies. Define a joint window $\mathcal{W}_H$ as an ellipsoid (or union) in observable space; apply the Bayesian/frequentist rules to the joint measure. Store correlations in `/predictions/.../attrs: C`.

Decision tables and audit trail

For each O, write to `/windows/kill`:

$$\{\texttt{obs_id},\ \texttt{bound_type},\ L_O,\ U_O,\ \texttt{confidence},\ p^{O}_{\text{kill}},\ p_O,\ \alpha_O,\ \texttt{decision} \in \{\texttt{proceed,hold,kill}\}\}.$$

Attach run hashes and dataset links to the exact posterior samples/likelihood surfaces used.

Worked template (minimal)

Observable	Type	Window	Metric	Rule	Decision key
$\sin^2\theta_W(M_Z)$	2-sided	$[L, U]$	p_{kill}, p	Bayes & Holm	`/windows/kill/thetaW`
$c_T(f)$	1-sided	$\leq 1 + \epsilon_G$	max ratio	Seq. Bayes	`/windows/kill/cT`
S, T, U	3D ellipsoid	$\in \mathcal{E}_{95}$	χ^2	α-spend	`/windows/kill/STU`

Acceptance.

Reproducible posterior summaries; decision tables. A release passes D if (i) posterior-based p^{O}_{kill} and frequentist p_O are recorded with seeds and code hashes; (ii) decisions match the predeclared rules for every O; (iii) sequential boundaries and spending functions are written and verified against the replay; (iv) `/windows/kill` contents are complete and unit-annotated.

Chapter 22

Governance, Risk, and Revision Playbook

22.1 Versioning, reviews, and data policies

Purpose.

This section defines how the USFT stack evolves without breaking *reproducibility*, *auditability*, or the declared DoV. We specify a versioning scheme spanning theory, code, data, and artifacts; we formalize review gates; and we state data policies (licensing, retention, access control, security).

Unified versioning: theory, code, and data

We adopt a *four-component* semantic versioning for public releases:

```
MAJOR.MINOR.PATCH+DATA
```

with the following meaning:

- **MAJOR**: breaking changes to axioms/definitions that alter predictions, data schemas, or acceptance gates; requires a new DoV declaration and re-validation of S/G/TQ/D.
- **MINOR**: backward-compatible additions (new observables, new solvers, tighter kill windows) that *do not* invalidate prior artifacts.
- **PATCH**: bug fixes and documentation clarifications that leave numerical outputs within $\varepsilon_{\text{replay}}$ tolerances.
- **DATA** (build metadata): monotone counter for the HDF5 bundle and notebooks; changes only when `msp_v7.h5` (or derivatives) are regenerated under identical code

but different random seeds or extended grids.

Each release is bound to: (i) a Git commit hash (theory & code), (ii) a container hash (runtime), (iii) a SHA256 manifest for the HDF5 bundle (see §4).

Branches and support windows.

We maintain `main` (bleeding edge), `release/x.y` (stabilization), and `lts/x.y` (long-term support with security & schema backports). Patches enter `lts` only if they preserve $\varepsilon_{\text{replay}}$ on all published benchmarks.

Review process and roles

All changes enter as merge requests (MRs) tagged by domain: `theory`, `numerics`, `inference`, `data-schema`, `docs`.

Checklist 22.1: Governance review checklist

- **Rationales & scope.** MR includes a problem statement, scope, and expected impact on predictions, windows, or DoV.
- **Gate impact.** Authors state whether S/G/TQ/Dcertificates are affected and attach re-run proofs if yes.
- **Reproducibility.** Replay on a minimal benchmark completes and matches within $\varepsilon_{\text{replay}}$ (recorded).
- **Security.** No elevation of access; secrets and keys handled via the vault; container immutability preserved.
- **Schema.** If `.h5` keys change, a migration note and unit tests are provided.
- **Docs.** Public-facing *Cards/UFO* and API docs updated atomically.

Roles.

Maintainer (merge authority), *Reviewer* (domain expert), *CI steward* (pipeline health), *Release manager*. At least two approvals are needed: one from the affected domain and one from CI steward when artifacts change.

Data policies: licensing, provenance, retention

Licensing.

Theory text and documentation: CC-BY-4.0. Code: permissive (Apache-2.0) unless solver dependencies require GPL. Data bundles (`.h5`): CC-BY-4.0 with explicit third-party attributions.

Provenance & signing.

Every bundle carries: (i) `schema_version`, `build_time_utc`; (ii) `git_commit`, `env_lock` (container digest); (iii) a `manifest_sha256` over all datasets; (iv) a detached `sig_ed25519` signature (release key). Provenance is verified in CI before distribution.

Access control.

Raw experimental inputs with usage restrictions are stored separately and referenced by content hashes. Public bundles contain only derived, license-clean products. User-level ACLs gate access to raw inputs; all releases are immutable.

Privacy & PII.

No personal data are stored. If collaboration metadata are included, they are hashed or anonymized. Logs redact credentials and network endpoints.

Retention.

LTS releases are retained for $\geq$ 36 months with containers and replay notebooks. Intermediate CI artifacts are kept for 90 days unless needed by a signed decision.

Release procedure (tagged, reproducible)

A release candidate (RC) is produced on `release/x.y` by the CI:

1. Build containers; run full gate suites; write `/audits` and `/windows/kill`.
2. Generate `msp_v7.h5` and replay notebooks; compute manifest hash and signature.
3. Create a *release note* including: version, DoV statement, changed windows, checksums, and known limitations.
4. Two-person review & freeze; tag `x.y.z+DATA`; publish bundle and docs.

Risk register and rollback

We maintain a risk ledger with fields: {`id`, `title`, `likelihood`, `impact`, `mitigation`, `trigger`}. Triggers include: gate regressions, replay drift $> \varepsilon_{\text{replay}}$, security advisories, and kill-window crossings by new data.

Checklist 22.2: Revision & rollback playbook

- **Detect.** CI dashboards flag any failed gate or replay drift.
- **Freeze.** Block merges to `release/lts` until triage completes.
- **Diagnose.** Bisect commits; reproduce locally with locked containers.
- **Mitigate.** Hotfix on `release/x.y` if PATCH; otherwise revert to last LTS.

- **Document.** Append postmortem; update Cards/UFO and schema notes.

Acceptance 22.1. Governance acceptance A release satisfies governance when: (i) versioning is consistent and signed; (ii) gate and replay certificates are present; (iii) data policies (license, provenance, ACL) are attached to the bundle; (iv) the risk register and rollback procedures are current and tested in CI.

22.2 Kill-event handling; rollback/patching

AR.

Decision logs; signed releases; rollback scripts.

Purpose.

This section prescribes the *operational* response when any observable crosses a predeclared kill window (§21.2) or a gate certificate regresses (S/G/TQ/D). The objectives are: (i) freeze and contain impact, (ii) perform reproducible triage, (iii) choose among KILL/HOLD/PATCH/ROLLBACK, (iv) re-release with signed artifacts and a permanent audit trail.

22.3 Single Source of Truth (SoT) Policy

Definition. The sole source of truth (SoT) for the operational stack is the spectral Lagrangian under `/action/terms`. All downstream numerical products must declare a dependency on it and carry its hash.

Commitments.

1. **Write-once:** `/action/terms` is immutable within a build.
2. **Canonicalization:** numeric blocks in `/action/terms` are serialized with deterministic ordering (see §24.1).
3. **Propagation:** every downstream group carries `action_hash` = root `lagrangian_id` and an explicit `depends_on` including `/action/terms`.
4. **Provenance:** all derivations are recorded under `/provenance/edges` and summarized in `graph_hash`.
5. **Versioning:** changes to `/action/terms` bump `lagrangian_id` and require a new public bundle tag.

Definitions and triggers

- **Kill event (KE).** A tuple KE $= (O, D, t, \texttt{run_id})$ where the decision rules of §21.2 imply KILL for observable O under data D at time t, with run metadata `run_id`.
- **Gate regression.** Any certified margin falls below zero: min `/cones/lmi/mu_min` $<$ 0 (G), min `/quantum/gksl/choi_eigs` $< -\varepsilon_{TQ}$ (TQ), or replay drift $> \varepsilon_{\text{replay}}$ in `/audits`.
- **Scope.** *Local* (affects one portfolio item) or *global* (affects the stack or DOV).

Freeze and triage pipeline (T0–T3)

On KE detection the CI opens an incident and executes T0–T3.

Checklist 22.3: Kill-event triage

- **T0 Freeze.** Lock merges to `release/lts`; tag current container and HDF5; archive logs.
- **T1 Reproduce.** Re-run the exact pipeline with locked `env_lock`, seeds, and inputs. Record $\widehat{O}$, posterior/likelihood surfaces, and decision outputs under `/audits/kill_events`.
- **T2 Validate data.** Check raw-data integrity (hashes, provenance), re-ingest if needed, and test for instrument/systematics flags. Mark KE as *instrumental* if the model never enters the kill region after validated corrections.
- **T3 Localize cause.** Bisect changes in theory/code/schema since last pass; compare predicted $O(\vartheta)$ to $\mathcal{W}_O$; estimate p^O_{kill} and frequentist p_O.

Decision rules (action selection)

Let $\mathcal{D}$ be the triage dossier (T0–T3). Choose the action:

A1. Proceed with Kill (science). If KE is *reproducible*, *non-instrumental*, and the model's predictions violate $\mathcal{W}_O$ with posterior $p^O_{\text{kill}} \geq \tau_{\text{kill}}$ or frequentist $p_O \leq \alpha_O$ (cf. §21.2), then KILL the affected hypothesis (or the current MSP* choice) within the declared DOV. Open a *revision program* (see below).

A2. Rollback (operational). If regression originates from a faulty change (`MAJOR`/`MINOR`/`PATCH`) and last LTS passes all gates, revert to the last LTS tag and republish artifacts; mark the faulty release as `yanked`.

A3. Hotfix/Patch. If KE is caused by a correctable defect (bug, missing correction, mis-specified window metadata) and does *not* change predictions beyond $\varepsilon_{\text{replay}}$, issue

`PATCH` with targeted fixes; re-sign artifacts.

A4. Hold/Collect. If triage is inconclusive (data under review, ambiguous systematics), freeze decision with a time-bounded collection plan and explicit additional data required.

Rollback and patching protocols

Rollback (to LTS).

(R1) Checkout last `lts/x.y` tag; rebuild containers; replay benchmarks.

(R2) Republish `msp_v7.h5` and notebooks; write `yanked_since=<tag>` in the yanked release.

(R3) Update public index and API to point consumers to the LTS bundle; keep the yanked bundle archived but not default.

Hotfix/Patch.

(P1) Create `hotfix/x.y.z` branch; implement minimal change; add regression tests.

(P2) Verify *no* change in declared windows and *no* drift $> \varepsilon_{\text{replay}}$.

(P3) Tag `x.y.(z+1)`; sign and publish; append to decision log.

Revision program (post-kill science)

When A1 applies, open a tracked revision with fields: {`id`, hypothesis removed, impacted chapters (Vols. I–VII), proposal for MSP*/inputs update, expected new windows, owners, review dates}. A revision may be *local* (observable-specific) or *global* (MSP*, DoV).

Forensics and audit trail

Record all KE artifacts in HDF5:

- `/audits/kill_events/<timestamp>/`

  `obs_id` (string), `decision` (`kill|hold|rollback|patch`), `p_kill` (float64), `p_value` (float64), `alpha` (float64), `posterior_hash`, `likelihood_hash`, `run_id`, `env_lock`, `git_commit`.

- `notes` (string): human-readable triage summary; `sig_ed25519`.

- Links to the exact windows used: `/windows/kill/<obs_id>`.

Decision-log template (YAML; also embedded in HDF5).

```
id: KE-2025-10-23-001
obs_id: sin2thetaW_MZ
decision: kill        # or hold/rollback/patch
```

```
rules:
  bayes: {p_kill: 0.973, tau_kill: 0.95}
  freq:  {p_value: 0.004, alpha: 0.01}
triage:
  reproducible: true
  instrumental: false
artifacts:
  git_commit: a1b2c3d4
  env_lock: sha256:...
  posterior_hash: sha256:...
  likelihood_hash: sha256:...
release:
  from: 1.7.2+42
  to:   1.7.3+43   # if patch
signatures:
  bundle_sig: ed25519:...
```

Automation hooks

- CI watches `/windows/kill` and `/audits`; on KE, it triggers (i) the triage workflow; (ii) merge freeze; (iii) notification to maintainers.
- The *rollback script* fetches last LTS artifacts, rewrites download aliases, regenerates API indices, and posts signed release notes.

Acceptance 22.2. Kill-handling acceptance A KE is *closed* when: (i) T0–T3 are documented in `/audits/kill_events`; (ii) the chosen action (kill/rollback/patch/hold) is executed and signed; (iii) public artifacts (HDF5, notebooks, Cards/UFO) are consistent with the decision; (iv) gates re-pass on the post-action bundle; (v) decision logs are immutable and reproducible.

Part VIII

Experimental Program and Probes

Chapter 23

Collider, Astrophysical, and Cosmological Probes

Aim. This part translates the USFT stack into concrete experimental probes. We specify observable vectors, the theory–data maps (including linearized oblique/EDM blocks), likelihoods with covariances, and *kill/hold* rules tied to S/G/TQ/D. The section below treats the electroweak (EW) precision set and permanent electric dipole moments (EDMs).

23.1 Electroweak and EDM experiments

Electroweak precision block (EWPO)

Let

$$\mathbf{O}_{\mathrm{EW}}^{\mathrm{exp}} \;=\; \Big(M_W,\ \sin^2\theta_{\mathrm{eff}}^{\ell},\ \Gamma_Z,\ \sigma_{\mathrm{had}}^{0},\ R_\ell,\ A_{\mathrm{FB}}^{b}\Big)^{\top}, \qquad \Sigma_{\mathrm{EW}} \ \text{ their experimental covariance.}$$

USFT predictions at the reference scale use the YM anchor (§18.1, §15.3) and vacuum–polarization corrections summarized by oblique parameters (S, T, U). Write the *linearized* theory–data map around the SM baseline $\mathbf{O}_{\mathrm{EW}}^{\mathrm{SM}}$:

$$\mathbf{O}_{\mathrm{EW}}^{\mathrm{th}} \;=\; \mathbf{O}_{\mathrm{EW}}^{\mathrm{SM}} \;+\; J_{\mathrm{EW}} \begin{pmatrix} S \\ T \\ U \end{pmatrix} \;+\; \Delta_{\mathrm{USFT}}, \tag{23.1}$$

where J_{EW} is the Jacobian (stored in HDF5) and Δ_{USFT} includes non-oblique shifts (e.g. altered Higgs/gauge couplings from the spectral sector). The oblique parameters are

defined by vacuum–polarization functions $\Pi_{XY}(q^2)$ in the usual way:

$$S = \frac{4s_W^2 c_W^2}{\alpha}\left[\Pi'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{s_W c_W}\Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0)\right], \tag{23.2}$$

$$T = \frac{1}{\alpha}\left[\frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}\right], \qquad U = \frac{4s_W^2}{\alpha}\left[\Pi'_{WW}(0) - c_W^2\Pi'_{ZZ}(0) - 2s_W c_W \Pi'_{Z\gamma}(0) - s_W^2\Pi'_{\gamma\gamma}(0)\right]. \tag{23.3}$$

Here $s_W^2 \equiv \sin^2\theta_W$, $c_W^2 = 1 - s_W^2$, and primes denote q^2-derivatives at 0.

Likelihood and decision rule.

$$\chi^2_{\mathrm{EW}} \;=\; \left(\mathbf{O}^{\mathrm{th}}_{\mathrm{EW}} - \mathbf{O}^{\mathrm{exp}}_{\mathrm{EW}}\right)^{\top}\Sigma^{-1}_{\mathrm{EW}}\left(\mathbf{O}^{\mathrm{th}}_{\mathrm{EW}} - \mathbf{O}^{\mathrm{exp}}_{\mathrm{EW}}\right). \tag{23.4}$$

A point passes the EW gate if $\chi^2_{\mathrm{EW}} \le \chi^2_{\mathrm{EW,\,gate}}$ (the quantile set under D). The *kill window* for this block is the complement of the acceptance ellipsoid.

HDF5 keys (EW).

- `/predictions/ewpo/theory` $\to \mathbf{O}^{\mathrm{th}}_{\mathrm{EW}}$, `units` and `axes` attached.
- `/data/ewpo/exp` $\to \mathbf{O}^{\mathrm{exp}}_{\mathrm{EW}}$, `/data/ewpo/cov` $\to \Sigma_{\mathrm{EW}}$.
- `/rg/flows/g123` and `/predictions/thetaW_MZ` feeding the map.
- `/maps/J_EW` $\to J_{\mathrm{EW}}$ (rows aligned with the observable list).

Notes on calibration.

When using $\sin^2\theta_W$ from the YM anchor at μ_{ref}, transport to M_Z with the recorded RG scheme (§15.3), then apply the oblique Jacobian to obtain $\sin^2\theta^{\ell}_{\mathrm{eff}}$ and M_W shifts.

Permanent EDMs (electron, nucleon, atoms)

Let

$$\mathbf{d}^{\mathrm{exp}} = \left(d_e^{\mathrm{UL}},\, d_n^{\mathrm{UL}},\, d_{\mathrm{Hg}}^{\mathrm{UL}},\, \ldots\right)^{\top}$$

denote current upper limits (converted to the same unit system) and $\mathbf{d}^{\mathrm{th}}$ the USFT predictions. We organize low-energy CP-odd sources as Wilson coefficients $\mathbf{C} = (C_e,\, C_q,\, \tilde{C}_g,\, \ldots)^{\top}$ for the EDM, chromo-EDM, and Weinberg operator, and write the *hadronic/atomic* map

$$\mathbf{d}^{\mathrm{th}} \;=\; \mathsf{R}_{\mathrm{EDM}}\,\mathbf{C} \;+\; \mathbf{d}_{\mathrm{floor}}, \tag{23.5}$$

where $\mathsf{R}_{\mathrm{EDM}}$ contains response coefficients (with nuisance ranges for hadronic/atomic matrix elements) and $\mathbf{d}_{\mathrm{floor}}$ is the *irreducible floor* predicted by USFT (e.g. two-loop Barr–Zee type) with uncertainty δ_{floor}.

One-sided decision rule (Gate D).

For each channel i,

$$\text{kill if} \qquad d_i^{\mathrm{UL}} \;<\; (1-\delta_i)\,|d_i^{\mathrm{th}}|, \qquad \delta_i \in [0,1] \text{ encodes theory/nuisance slack.} \tag{23.6}$$

Otherwise, keep and update posteriors on $\mathbf{C}$ (or on the spectral parameters feeding $\mathbf{C}$).

HDF5 keys (EDM).

- `/predictions/edm/theory` $\to \mathbf{d}^{\mathrm{th}}$, `/predictions/edm/floor` $\to \mathbf{d}_{\mathrm{floor}}$.
- `/data/edm/exp` $\to \mathbf{d}^{\mathrm{exp}}$, `/data/edm/slack` $\to \{\delta_i\}$.
- `/maps/R_EDM` $\to \mathsf{R}_{\mathrm{EDM}}$ with `axes` documenting (channel, Wilson).

Operational remarks.

(i) Treat atomic/nuclear uncertainties as nuisance priors and profile or marginalize in the likelihood; (ii) when several EDMs constrain the same $\mathbf{C}$, combine them with a block-diagonal covariance (or full correlations if provided); (iii) publish the floor vector and the one-sided windows in `/windows/kill/edm_*`.

Acceptance 23.1. EW & EDM acceptance *Pass* if (a) χ^2_{EW} lies inside the declared $\mathsf{Dellipsoid}$ and (b) all EDM channels satisfy the one-sided rule (23.6) with documented δ_i. The HDF5 bundle must contain the entries listed above, with unit/axis metadata and hashes.

23.2 Neutrino long-baseline; DI regions

Aim.

Operationalize long–baseline appearance/disappearance channels and attach *device–independent* (DI) feasibility tests that are robust to colored noise and calibration slacks. We give (i) the forward model from USFT parameters to binned counts, (ii) the NSI map and oscillation Hamiltonian on (M, g_{eff}), (iii) likelihoods with ND–FD cancellation, and (iv) DI constraints and finite–statistics concentration bounds.

USFT $\to$ NSI Hamiltonian

On a background with effective metric g_{eff} and electron density $n_e(x)$, the flavor–basis Hamiltonian for neutrinos of energy E is

$$H_\nu(E;x) = \frac{1}{2E}\,U\,\mathrm{diag}(m_1^2, m_2^2, m_3^2)\,U^\dagger + \underbrace{\sqrt{2}\,G_F\,n_e(x)}_{A(x)}\Big(\mathbf{1}_{ee} + \varepsilon\Big), \tag{23.7}$$

where U is the PMNS matrix, $\mathbf{1}_{ee}$ projects onto ν_e, and $\varepsilon = (\varepsilon_{\alpha\beta})$ is a Hermitian NSI matrix (including complex off–diagonals). USFT parameters $(\rho, \eta, \ldots)$ enter through an

effective map

$$\varepsilon_{\alpha\beta} \;=\; \mathsf{M}_{\alpha\beta}(\theta_{\mathrm{USFT}};\, E, \mathrm{env}) \qquad \Big(\text{stored under } \texttt{/maps/nsi/M}\Big), \tag{23.8}$$

with "env" encoding baseline geology, ionization fraction, and colored–noise slacks in the meter layer. Probabilities follow from H_ν (and $H_{\bar\nu}$ with $A \to -A$ and $\varepsilon \to \varepsilon^*$) either by direct diagonalization or a Magnus/Trotter integrator on the Cauchy slice.

Forward model (ND–FD)

Let $D \in \{N, F\}$ label near/far detectors, i an energy bin, and $\alpha \to \beta \in \{\mu \to e, \mu \to \mu, \ldots\}$ a channel. The expected counts are

$$\lambda^{D,i}_{\alpha\beta} = \underbrace{\mathcal{E}^{D,i}\, \Phi^{D,i}_{\alpha}\, \sigma^{D,i}_{\beta}\, \epsilon^{D,i}_{\beta}}_{T^{D,i}_{\beta}}\; P_{\alpha\beta}(E_i; L_D;\, \varepsilon, \mathrm{PMNS}) \;+\; b^{D,i}_{\alpha\beta}, \tag{23.9}$$

where Φ is flux, σ cross section, ϵ efficiency, $\mathcal{E}$ exposure, and b backgrounds (all with nuisance parameters). The ND–FD transfer cancels shared systematics by a linear map

$$\widehat{\lambda}^{F,i}_{\alpha\beta} = \sum_j \mathsf{R}^{ij}_{\alpha\beta}\, n^{N,j}_{\alpha\beta}\;, \qquad \mathsf{R}^{ij} \text{ precalibrated from MC/control samples, stored in HDF5.} \tag{23.10}$$

Likelihood with pulls.

Using Poisson terms and Gaussian pulls for nuisances η,

$$\chi^2_{\mathrm{NDFD}}(\theta, \eta) = 2 \sum_{D,i,\alpha\beta} \left[\lambda^{D,i}_{\alpha\beta} - n^{D,i}_{\alpha\beta} + n^{D,i}_{\alpha\beta} \ln \frac{n^{D,i}_{\alpha\beta}}{\lambda^{D,i}_{\alpha\beta}}\right] + (\eta - \bar\eta)^\top W^{-1} (\eta - \bar\eta), \tag{23.11}$$

$$\chi^2_{\mathrm{map}} = \Big(\widehat{\lambda}^F - n^F\Big)^\top \Sigma_F^{-1} \Big(\widehat{\lambda}^F - n^F\Big), \tag{23.12}$$

and the working objective is $\chi^2_{\mathrm{LBL}} = \chi^2_{\mathrm{NDFD}} + \chi^2_{\mathrm{map}}$. Gate Dattaches the decision windows to the minimum χ^2 after profiling in η.

DI feasibility under colored slacks

We build a two–setting, two–outcome coarse–graining from experimental toggles (e.g. horn polarity/beam tune and detector PID threshold), yielding empirical conditional distributions $p(o|x, y)$ with $o \in \{\pm 1\}$, $x, y \in \{0, 1\}$.

(i) No–signalling residuals.

$$\Delta_{\mathrm{NS}} = \max_{x, y \neq y'} \left| \sum_o p(o|x, y) - \sum_o p(o|x, y') \right|, \qquad \text{accept if } \; \Delta_{\mathrm{NS}} \le \delta_{\mathrm{NS}}. \tag{23.13}$$

(ii) CHSH/Tsirelson with colored slacks.

Define $E_{xy} = \sum_{o=\pm 1} o\, p(o|x,y)$ and $S = E_{00} + E_{01} + E_{10} - E_{11}$. With colored embedding (§??) we allow a small margin Δ_{col} (`/di/margins/delta_col`):

$$|S| \;\le\; 2\sqrt{2} + \Delta_{\text{col}}, \qquad \text{accept if satisfied in all analysis bins.} \tag{23.14}$$

Finite–statistics concentration.

For N trials per (x,y), Hoeffding implies

$$\left|\widehat{p} - p\right| \le \sqrt{\frac{\ln(2/\delta)}{2N}} =: r(N,\delta), \qquad \Rightarrow \qquad |\,\widehat{S} - S\,| \le 4\, r(N,\delta). \tag{23.15}$$

We therefore test (23.14) with a widened band $2\sqrt{2} + \Delta_{\text{col}} + 4\, r(N,\delta)$ at the chosen confidence δ.

HDF5 keys (neutrino block).

- `/neutrino/coords`: E-bin edges, baselines L, detector labels.
- `/neutrino/flux`, `/neutrino/xsec`, `/neutrino/eff`, `/neutrino/exposure`.
- `/neutrino/theory_P` $\to P_{\alpha\beta}(E_i, L_D)$; `/maps/nsi/M` for $\varepsilon(\theta_{\text{USFT}})$.
- `/neutrino/events/near`, `/neutrino/events/far`, `/neutrino/R_N2F` for the transfer matrix R.
- `/di/p_hat_(x,y)`, `/di/NS_residual`, `/di/CHSH_S`, `/di/margins/{delta_col,delta_NS}`, plus N per cell.
- `/windows/kill/neutrino`: per–observable windows and p–value thresholds.

Acceptance 23.2. LBL/DI acceptance *Pass* if (a) the profiled χ^2_{LBL} is inside the declared Dwindow; (b) ND–FD transfer residuals are consistent (χ^2_{map} below threshold); (c) DI tests satisfy (23.13) and (23.14) at confidence δ with the finite–statistics widening (23.15). Otherwise issue a *kill* with the corresponding artifact written under `/audits/di`.

23.3 GW/FRB/CMB/LSS links

Aim.

Build a *multi–messenger* bridge from USFT parameters θ_{USFT} to jointly–constrained observables across gravitational waves (GW), fast radio bursts (FRB), the CMB, and large–scale structure (LSS). We provide (i) timing/dispersion relations along the effective cone g_{eff}, (ii) lensing and anisotropy cross–spectra, (iii) a joint likelihood with *orthogonalized* templates to break degeneracies, and (iv) HDF5 keys for reproducibility.

Time–of–flight and dispersion along $g_{\rm eff}$

For a messenger $X \in \{\mathrm{GW}, \mathrm{EM}, \nu\}$ with phase/group speed $c_X(f,z)$, the excess time of flight from source redshift z_s is

$$\Delta t_X(f; z_s) = \int_0^{z_s} \frac{1+z}{H(z)} \left(\frac{1}{c_X(f,z)} - \frac{1}{\mathrm{cEff}(z)} \right) dz, \qquad \mathrm{cEff}(z) \text{ from } g_{\rm eff}. \tag{23.16}$$

For a GW/EM coincidence, the test statistic uses $\Delta t_{\mathrm{GW-EM}} = \Delta t_{\mathrm{GW}} - \Delta t_{\mathrm{EM}}$. The USFT mapping $c_T(f, z; \theta_{\mathrm{USFT}})$ (from ρ, η and mixing to tensor modes) feeds Δt_{GW}.

FRB dispersion measure (DM).

With comoving electron density $n_e(z)$ and possible $g_{\rm eff}$–induced modifiers $\delta_{\mathrm{DM}}(z; \theta_{\mathrm{USFT}})$,

$$\mathrm{DM}(z_s) = \int_0^{z_s} \frac{n_e(z)}{(1+z)\, H(z)} \left[1 + \delta_{\mathrm{DM}}(z; \theta_{\mathrm{USFT}}) \right] dz. \tag{23.17}$$

Joint GW–FRB (or GW–GRB) events constrain $(c_T - 1, \delta_{\mathrm{DM}})$ with cosmic–variance limited priors on $n_e(z)$.

Lensing/ISW and anisotropy cross–spectra

Let Φ, Ψ be the metric potentials in Newtonian gauge with an effective Poisson law $k^2 \Psi = -4\pi G_{\rm eff}(k,z)\, a^2 \rho_m \Delta$. We form standard angular spectra and their USFT–deformed kernels:

$$C_\ell^{\kappa g} = \int_0^{\chi_*} \frac{d\chi}{\chi^2}\, W_\kappa(\chi)\, W_g(\chi)\, P_\delta\!\left(\frac{\ell + 1/2}{\chi}, z(\chi) \right), \tag{23.18}$$

$$C_\ell^{Tg}\ (\mathrm{ISW}) = \frac{2}{\pi} \int k^2 dk\, \Theta_\ell^{\mathrm{ISW}}(k)\, \Delta_\ell^g(k)\, P_{\mathcal{R}}(k), \qquad \Theta_\ell^{\mathrm{ISW}} \propto \partial_\eta(\Phi + \Psi), \tag{23.19}$$

where P_δ and $\partial_\eta(\Phi + \Psi)$ are computed with $G_{\rm eff}$, η–relaxation, and cone–feasible sound speeds from USFT. Statistical anisotropy enters via a quadrupolar template $\Delta P/P \equiv g_*(k)\, (\hat{\mathbf{k}} \cdot \hat{\chi})^2$ that propagates to $\{C_\ell^{TT}, C_\ell^{EE}, C_\ell^{\kappa\kappa}, \ldots\}$.

Joint likelihood and orthogonal templates

Collect binned data

$$\mathbf{d} = \left(\Delta t_{\mathrm{GW-EM}},\ \mathrm{DM},\ C_\ell^{Tg},\ C_\ell^{\kappa g},\ C_\ell^{\kappa\kappa},\ P(k,z), \ldots \right)^\top$$

with covariance $\mathbf{C}$ (including shared cosmic variance and selection effects). The Gaussian log–likelihood is

$$-2 \ln \mathcal{L}(\theta) = (\mathbf{d} - \mathbf{m}(\theta))^\top \mathbf{C}^{-1} (\mathbf{d} - \mathbf{m}(\theta)) + \ln |\mathbf{C}| + \text{const.} \tag{23.20}$$

To reduce parameter degeneracies we build *orthogonal* templates $\{\mathbf{t}_a\}$ for the linear response $\partial\mathbf{m}/\partial\vartheta_a$ by Gram–Schmidt under the inner product $\langle\mathbf{u},\mathbf{v}\rangle = \mathbf{u}^\top\mathbf{C}^{-1}\mathbf{v}$. We report the *condition number* κ_T of the template Fisher block as a diagnostic; κ_T must improve (decrease) when cross–probe terms are added (orthogonality gate).

Selection and timing systematics

Selection functions $S_X(z, \mathrm{SNR})$ (for GW, FRB, galaxy samples) enter $\mathbf{C}$ and $\mathbf{m}$. Timing systematics are modeled as $\Delta t \to \Delta t + \delta t_{\mathrm{clock}} + \delta t_{\mathrm{host}}$ with priors stored in HDF5 and marginalized in the sampler. Cone feasibility is enforced by requiring $c_T(f,z) \leq \mathrm{cEff}(z)\,[1+\varepsilon_G]$ (cf. §17.1).

HDF5 keys (multi–probe).

- `/astro/coords`: z, f, ℓ, k, and survey masks.
- `/astro/timing`: $c_T(f,z)$, $\Delta t_{\mathrm{GW-EM}}(z)$, priors for $\delta t_{\mathrm{clock}}, \delta t_{\mathrm{host}}$.
- `/astro/frb`: DM tables, $n_e(z)$, $\delta_{\mathrm{DM}}(z)$.
- `/astro/spectra`: $C_\ell^{\kappa g}, C_\ell^{Tg}, C_\ell^{\kappa\kappa}$, kernels, and masks.
- `/astro/lss`: $P(k,z)$, growth $D(z)$, $G_{\mathrm{eff}}(k,z)$.
- `/astro/templates`: orthogonal templates $\mathbf{t}_a$ and template Fisher, with `cond_number`.
- `/astro/like`: covariance $\mathbf{C}$, data vector $\mathbf{d}$, and likelihood options.

Acceptance 23.3. Multi–probe acceptance **Protocol specs** are shipped for each probe (selection, masks, timing priors); **likelihoods** implement (23.20) with documented $\mathbf{C}$ and reproduce known baselines on GR mock data; **orthogonal templates** reduce the template Fisher condition number (κ_T decreases when adding cross–probe terms) and pass the cone constraint $c_T \leq \mathrm{cEff}(1+\varepsilon_G)$ on the declared DoV.

Part IX

Automation, Data Schema, and Reproducibility

Chapter 24

Data Schema and Manifests

Aim.

This chapter specifies the public `HDF5` schema (`v7`) that the USFT toolchain writes and reads. The schema makes the stack auditable and *reproducible*: every numeric dataset carries units and axes; every group declares provenance; every result ships with gates (S/G/TQ/D) and a DoV audit.

Purpose and scope

- **Rebuildability.** One–command replay regenerates all figures, tables, and gate certificates from a single `.h5` file.

- **Auditability.** Each artifact has a hash and manifest; integrity checks verify units, axes, and constraints.

- **Interoperability.** Keys and shapes are stable; optional blocks are feature–gated via attributes.

Top-level attributes (root "/")

Attribute	Type	Meaning
`schema_version`	string	Must equal `"v7"`.
`build_time_utc`	string (ISO 8601)	File creation time (UTC).
`git_commit`	string	Source revision hash.
`env_lock`	string	Environment lockfile hash (e.g., Conda/Poetry).
`generator`	string	Pipeline name (e.g., `usft-v7`).
`units_system`	string	`"SI"` or `"natural(c=hbar=1)"`.
`lagrangian_id`	string (hex SHA256)	Hash over `/action/terms` (single source of truth for the spectral Lagrangian).
`graph_hash`	string (hex SHA256)	Rolled hash over `/provenance/edges`+`/provenance/hashes` (consistency of the build graph).

24.1 Source of Truth (SoT) specification

Root attributes (recall).

The file root stores `lagrangian_id` (SHA256 over `/action/terms`) and `graph_hash` (rolled SHA256 over `/provenance/*`).

Canonical hash for `/action/terms`.

Compute `lagrangian_id` as follows:

1. Depth-first, lexicographic traversal of all datasets under `/action/terms`.
2. For each dataset, append to a byte buffer: path (UTF-8), dtype string, shape, `units` and `axes` attributes, then raw bytes in native endianness (float64 unless otherwise declared).
3. Exclude volatile attributes (`build_time_utc`, environment).
4. Hash the buffer with SHA256; store hex string in root `lagrangian_id`.

Graph hash `graph_hash`.

Build a buffer from `/provenance/hashes` and `/provenance/edges` sorted lexicographically by path; hash with SHA256; store in root.

Downstream enforcement.

Every numeric dataset outside `/action/terms` must carry: `units`, `axes`, `depends_on` (including `/action/terms`), and `action_hash` identical to root `lagrangian_id`.

Global conventions

- **Units.** Every numeric dataset/group has a `units` attribute.
- **Axes.** Every array has an `axes` attribute (ordered list), with coordinate arrays present under `/coords`.
- **Types.** `float64` for reals; `int32/int64` for indices; booleans as `uint8` (0/1).
- **Provenance.** Each top–level group carries `hash_sha256`.

24.2 Unified HDF5 v7 Schema and Manifests (Final)

24.2.1 Top-level attributes (root /)

The file root `/` *must* contain:

- `schema_version` = `"v7"` (string);
- `build_time_utc` (ISO-8601 string), `git_commit`, `env_lock`;
- `units_system` = `"SI"` or `"natural(c=hbar=1)"`;
- `lagrangian_id` (SHA256 over `/action/terms`);
- `graph_hash` (rolled SHA256 over the provenance graph, §24.2.5).

24.2.2 Directory layout (final)

```
/  (attrs: schema_version, build_time_utc, git_commit, env_lock, units_system,
           lagrangian_id, graph_hash)

  /action/
    terms/             # spectral Lagrangian blocks (kinetic, potential, mixings, ray
    linearization/     # background-indexed perturbation matrices (axes required)
      K                # K(a,k,theta,...)    units: --
      Z                # Z(a,k,theta,...)    units: --
      M2               # M^2(a,k,theta,...)  units: energy^2
    report_yaml        # short narrative report (string dataset)

  /rg/
    flows              # beta tables and g123(mu) tracks
    scheme             # scheme metadata (MSbar/MOM, loop order)
    thresholds         # decoupling thresholds and validity ranges
```

```
/sm_map/
  K                  # 3x3 kinetic Gram from YM anchor
  O                  # 3x3 orthogonal diagonalizer (if mixing present)
  alpha_tilde        # diag entries after K = O^T diag(alpha_tilde) O
  nY_convention      # {"SM","GUT(5/3)"} (string)
  g123_ref           # (g1,g2,g3) at mu_ref
  thetaW_ref         # sin^2(theta_W) at mu_ref
  charges            # table of matter reps (rep, Y, SU2, SU3, multiplicity)
  anomalies          # cancellation checklist / residuals
  status             # {pass|warn|fail} for SM matching

/backgrounds/
  minkowski/         # flat background sheets/params
  frw/               # FRW tables: H(a), w_eff(a), g_eff, ...
  bh_stationary/     # stationary BH backgrounds
  params_yaml        # chosen background settings (string dataset)

/cones/
  lmi/
    mu_min           # min LMI eigenvalue grids
    L_loc            # locality/causality radius
    v_front          # front speeds vs c_eff

/quantum/
  gksl/              # local Lindblad rates, modes
  choi/              # Choi spectra and CPTP diagnostics
  logs/              # quantum verification logs (Hadamard/GNS, no-signalling)

/predictions/
  gw/                # c_T(f,z), dispersion, attenuation      (attr: obs_id="cT")
  edm/               # EDM floors and predictions             (attr: obs_id="EDM_*"
  nsi/               # NSI matrices                           (attr: obs_id="NSI_*"
  astro/             # FRB/CMB/LSS links                      (attr: obs_id="ASTRO_

/windows/
  kill/              # falsification windows (link by obs_id to /predictions/*)
  envelopes/         # confidence/credible envelopes per observable
```

```
/maps/
  R_EDM              # mapping matrices: params -> EDM observables
  NSI_M              # NSI maps: fields -> effective ops
  geom/              # geometric/topological maps used by cones/predictions

/neutrino/
  LBL/               # ND->FD transfer matrices and likelihoods
  DI/                # device-independent bounds (CHSH/Bell) with margins
  priors/            # priors for neutrino-sector parameters

/astro/
  templates/         # orthogonal templates used in forecasts
  meta/              # survey/time metadata

/coords/
  axes/              # coordinate arrays {a, k, theta, f, mu, ...}
  frames/            # frame definitions (metadata)

/audits/
  gates/             # unified gate results with stage tags (see below)
  dov_tiling/        # DoV tiling + boundary margins
  manifest_hash      # cryptographic hash of the manifest subset

/provenance/
  edges              # (from_path, to_path, reason, tool, version, time_utc)
  hashes             # (path, sha256) for major nodes

/bundle/
  replay/            # one-command replay script + checksums
  env_yaml           # environment lock (string dataset)

/runs/
  inputs/            # YAML/JSON configs (as string datasets)
  logs/              # time-stamped execution logs
```

24.2.3 Standard dataset attributes

All numeric datasets must carry:

- `units` (string) and `axes` (ordered list of axis names, each present under `/coords/axes`);

- `depends_on` (list of HDF5 paths) listing upstream sources;
- `action_hash` (SHA256 copied from root `lagrangian_id`) for all downstream groups.

All `/predictions/*` datasets must also carry:

- `obs_id` (string) matching an entry in `/windows/kill`.

24.2.4 Unified gates with stage tags

All gate results live under `/audits/gates` as a tabular dataset with columns:

`gate` (S|G|TQ|D), `stage` (`"action"|"rg"|"sm"|"quantum"|"predictions"`), `pass` (uint8), `metrics_path` (string).

No gate datasets appear anywhere else (e.g. no `action/gates`).

24.2.5 Provenance graph

/provenance/edges is a table `(from_path, to_path, reason, tool, version, time_utc)`, and **/provenance/hashes** is a table `(path, sha256)`. A rolled hash over all edges/hashes is stored at root as `graph_hash`. Every downstream group (e.g. `/predictions/gw`) *must* list `depends_on` and copy `action_hash`.

24.2.6 Integrity constraints (final)

1. **Units & axes**: every numeric dataset has `units` and `axes`; each axis exists under `/coords/axes`.
2. **Cones & causality**: if `/cones/lmi/v_front` exists then $\max(v_{\text{front}}/c_{\text{eff}}) \leq 1 + \varepsilon_G$; `mu_min` ≥ 0 on the declared DoV.
3. **SM anchor completeness**: `/sm_map` contains `K`, `O` (orthogonal), `alpha_tilde` (positive), `nY_convention`, `g123_ref`, `thetaW_ref`, `charges`, `anomalies`.
4. **Anomaly gate (SM stage).** Dataset `/sm_map/anomalies` exists with fields `residuals`[4], `norm_score`, `eps_A`, `pass` (uint8), and table `/sm_map/charges/table` is non-empty. Require $\|\texttt{residuals}\|_\infty \leq$ `eps_A` and `norm_score` $\leq$ `eps_A`. Also, `/audits/gates` contains a row with `gate="S"`, `stage="sm"`, `pass=1`, `metrics_path="/sm_map/an`
5. **RG linkage**: `/rg/scheme` and `/rg/flows` exist and `/sm_map/g123_ref` is consistent with `/rg/flows` at μ_{ref} within tolerance.
6. **Predictions–windows binding**: every `/predictions/*` has `obs_id`; every `obs_id` appears in `/windows/kill` with valid `bound_type` $\in$ {one-sided,two-sided} and `lower` $\leq$ `upper`.

7. **Constraint-closure (Gate S, stage="action").** Dataset `/audits/constraint_closure` exists with fields `rho_br`, `rho_Jac`, `eps_abs`, `eps_rel`, `pass`. Require `pass`=1 and a corresponding row in `/audits/gates` with `gate="S"`, `stage="action"`, `metrics_path="/audits/`

8. **Gates centralization**: all gate results are under `/audits/gates` with valid `stage`. No duplicates elsewhere.

9. **Provenance coverage**: each downstream node has `depends_on` and `action_hash`; `/provenance/edges` includes edges for all such dependencies; `graph_hash` matches recomputed hash.

10. **Reproducibility**: `/bundle/replay` and `/runs/inputs` exist; an independent replay reproduces key metrics within $\varepsilon_{\text{replay}}$.

11. **RG linkage.** `/rg/scheme` and `/rg/thresholds` exist. `/rg/linkage` exists with fields `g123_flows`, `g123_ref`, `residuals`, `eps_abs`, `eps_rel`, `score`, `pass`. Require `pass`=1 and a corresponding row in `/audits/gates` with `gate="S"`, `stage="rg"`.

12. **Single source-of-truth.** Root `lagrangian_id` equals the recomputed SHA256 over `/action/terms` (canonical order). Every downstream dataset carries `action_hash` identical to `lagrangian_id` and lists `/action/terms` in `depends_on`. `/audits/sot` exists with `pass`=1 and a gate record under `/audits/gates` with `gate="S"`, `stage="action"`.

24.2.7 Acceptance (Data schema v7)

The file passes the schema gate iff all Integrity constraints (1–8) hold and `/audits/gates` contains stage-tagged passes for S, G, TQ, and D relevant to the shipped predictions.

Notes for YAML mirrors.

Any `*.yaml` snapshot stored as a string dataset (`report_yaml`, `params_yaml`, `env_yaml`) must be mirrored by structured numeric/scalar datasets where applicable (e.g. the entries of `/rg/scheme`, `/backgrounds/*/params`, `/runs/inputs`).

1. **Units & axes.** Every numeric dataset carries attributes `units` (string) and `axes` (ordered list). Each axis listed exists under `/coords/axes`.

2. **Root attributes.** The root `/` has `schema_version="v7"`, `build_time_utc`, `git_commit`, `env_lock`, `units_system`, `lagrangian_id` (SHA256 of `/action/terms`), and `graph_hash` (rolled SHA256 over `/provenance/*`). Recompute both hashes and require exact match.

3. **Cones & causality (Gate G).** If `/cones/lmi/v_front` exists then $\max(v_{\text{front}}/c_{\text{eff}}) \leq 1 + \varepsilon_G$ on the declared DoV; datasets `mu_min` are ≥ 0; `L_loc` > 0.

4. **SM anchor completeness.** `/sm_map` contains `K`, `O`, `alpha_tilde`, `nY_convention`, `g123_ref`, `thetaW_ref`, `charges`, `anomalies`; require $K \succ 0$ and all entries of `alpha_tilde`> 0.

5. **RG linkage.** `/rg/scheme` and `/rg/flows` exist. Values in `/sm_map/g123_ref` equal the tracks in `/rg/flows` at μ_{ref} within tolerance ε_{RG} (recorded as an attribute).

6. **Predictions–windows binding (Gate D).** Every dataset under `/predictions/*` has an attribute `obs_id` that appears in `/windows/kill`. For each record, `bound_type` $\in$ {`one-sided`, `two-sided`}, `lower` $\leq$ `upper`, and `confidence` $\in (0, 1]$.

7. **Gates centralization.** All gate outcomes live *only* under `/audits/gates` with columns {`gate=S|G|TQ|D`, `stage="action|rg|sm|quantum|predictions"`, `pass` (0/1), `metrics_path`}. No gate datasets appear elsewhere.

8. **Provenance coverage.** Each downstream group carries attributes `depends_on` (list of HDF5 paths) and `action_hash` (copy of root `lagrangian_id`). `/provenance/edges` has a row for every declared dependency; recomputed rolled hash equals root `graph_hash`.

9. **YAML mirrors.** Any `*.yaml` string dataset (`report_yaml`, `params_yaml`, `env_yaml`) is mirrored by structured numeric/scalar datasets where applicable (e.g. entries of `/rg/scheme`, `/backgrounds/*/params`, `/runs/inputs`).

10. **Reproducibility.** `/bundle/replay` and `/runs/inputs` exist; an independent replay reproduces key metrics (GCI, energies, c_T, $g_{1,2,3}$ at $M_Z \ldots$) within `$ε_{replay}`.

`/action/terms`

Key	Type	Units	Axes	Description
`kinetic`	`float64[...]`	energy density	[a,k,theta]	Kinetic/metric blocks (e.g., K,Z tensors).
`potential`	`float64[...]`	energy density	[a]	$V(\Phi, \rho, \eta, T, \chi)$ on backgrounds.
`mixings`	`float64[...]`	energy density	[a,k,theta]	Cross-couplings among descriptors.
`rayleigh`	`float64[...]`	power density	[a]	Rayleigh dissipation functional (Gate D).

Key	Type	Units	Axes	Description
K	float64[...]	–	[a,k,theta]	Perturbation kinetic matrix (ghost-free test).
Z	float64[...]	–	[a,k,theta]	Symplectic/flux matrix (cones, group velocity).
M2	float64[...]	energy^2	[a,k,theta]	Mass/coupling blocks; gap maps and thresholds.

Key	Type	Units	Axes	Description
flows.mu	float64[n_mu]	energy	[mu]	RG scale grid.
flows.beta	float64[n_mu,n_g]	–	[mu,g]	Beta functions for g_i.
flows.g123	float64[n_mu,3]	–	[mu,i]	Tracks (g_1, g_2, g_3).
scheme	struct/string	–	–	Scheme name + loop order (e.g., $\overline{\text{MS}}$/MOM).
thresholds	float64[...]	energy	[species]	Decoupling masses/scales + validity flags.

/action/linearization

/rg

/sm_map

/backgrounds

/cones/lmi

/quantum

/predictions

All prediction datasets must carry `obs_id` *matching an entry in* `/windows/kill`*.*

Key	Type	Units	Axes	Description
K	float64[3,3]	–	[I,J]	3×3 kinetic Gram from YM anchor.
O	float64[3,3]	–	[I,J]	Orthogonal diagonalizer of K (mixing removal).
alpha_tilde	float64[3]	–	[I]	Diagonal entries after $K = O^\top \mathrm{diag}(\alpha)\, O$.
nY_convention	string	–	–	{SM, GUT(5/3)}.
g123_ref	float64[3]	–	[i]	(g_1, g_2, g_3) at μ_{ref}.
thetaW_ref	float64	–	–	$\sin^2\theta_W(\mu_{\mathrm{ref}})$.
charges	table	–	–	Matter reps $(\mathrm{rep}, Y, SU2, SU3, \mathrm{mult})$.
anomalies	table	–	–	Residuals for $[SU(3)]^2U(1)$, $[SU(2)]^2U(1)$, $[U(1)]^3$, grav^2–$U(1)$.
status	string	–	–	{pass|warn|fail} for SM matching.

Key	Type	Units	Axes	Description
minkowski/params	struct	–	–	Flat background scalars (refs, gauges).
frw/H	float64[n_a]	1/time	[a]	Hubble $H(a)$ tables.
frw/w_eff	float64[n_a]	–	[a]	Effective equation-of-state.
frw/g_eff	float64[...]	–	[a]	Effective metric components (as needed).
bh_stationary/params	struct	–	–	Stationary BH parameters.
params_yaml	string	–	–	Snapshot of chosen background settings.

Key	Type	Units	Axes	Description
mu_min	float64[...]	–	[a,k,theta]	Minimal LMI eigenvalue grids (Gate G).
L_loc	float64[...]	length	[a,k]	Locality/causality radius.
v_front	float64[...]	speed	[a,k,theta]	Front speeds vs. c_{eff} (causality).

Key	Type	Units	Axes	Description
gksl/rates	float64[...]	1/time	[modes]	Lindblad rates (local GKSL).
gksl/modes	float64[...]	–	[space]	Mode shapes / support info.
choi/eigs	float64[n]	–	[n]	Choi eigen-spectrum (CPTP margins).
choi/metrics	float64[...]	–	[tests]	Trace, contractivity, DI bounds.
logs	string	–	–	Quantum verification logs (Hadamard/GNS, no-signalling).

Key (attr: obs_id)	Type	Units	Axes	Description
gw/cT	float64[n_f]	–	$[f]$	GW speed $c_T(f, z)$; attr obs_id="cT".
edm/vector	float64[n_EDM]	e·cm	[species]	EDM predictions/floors (electron, neutron, nuclei).
nsi/epsilon	float64[...]	–	$[\alpha, \beta, f]$	NSI matrices $\varepsilon^f_{\alpha\beta}$.
astro/templates	float64[...]	–	$[k, \ell]$	Orthogonal templates for FRB/CMB/LSS.
ew/thetaW_MZ	float64	–	–	Weak mixing angle $\sin^2\theta_W$ at M_Z; attr obs_id="thetaW_MZ".

Key	Type	Units	Axes	Description
kill/obs_id	string	–	–	Observable id linking to /predictions/*.
kill/bound_type	string	–	–	{one-sided|two-sided}.
kill/lower	float64	[obs]	–	Lower bound of kill/decision window.
kill/upper	float64	[obs]	–	Upper bound of kill/decision window.
kill/confidence	float64	–	–	Confidence level (e.g., 0.95).
envelopes	float64[...]	[obs]	[grid]	Confidence/credible envelopes per observable.
kill/obs_id	string	–	–	thetaW_MZ
kill/bound_type	string	–	–	two-sided
kill/lower	float64	–	–	0.2305
kill/upper	float64	–	–	0.2320
kill/confidence	float64	–	–	0.95

Key	Type	Units	Axes	Description
R_EDM	float64[...]	–	[params,obs]	Mapping: Lagrangian params → EDM observables.
NSI_M	float64[...]	–	[fields,ops]	NSI map: fields → effective operators.
geom/..	float64[...]	–	[space]	Geometric/topological maps for cones/predictions.

Key	Type	Units	Axes	Description
LBL/transfer	float64[...]	–	[E,channel]	ND→FD transfer matrices / appearance channels.
LBL/like	float64[...]	–	[params]	Likelihood surfaces.
DI/bounds	float64[...]	–	[test]	Device-independent bounds (CHSH/Bell).
priors	float64[...]	–	[params]	Priors for neutrino-sector parameters.

Key	Type	Units	Axes	Description
templates	float64[...]	–	$[k, \ell]$	Orthogonal templates used in forecasts.
meta	struct/string	–	–	Survey IDs, exposure, timing metadata.

Key	Type	Units	Axes	Description
axes/a	float64[n_a]	–	[a]	Scale factor.
axes/k	float64[n_k]	1/length	[k]	Wavenumber.
axes/theta	float64[n_theta]	rad	[θ]	Polar angle w.r.t. $\hat{\chi}$.
axes/f	float64[n_f]	Hz	[f]	GW frequency.
axes/mu	float64[n_mu]	energy	[μ]	RG scale.
frames	struct/string	–	–	Frame definitions and notes.

Key	Type	Units	Axes	Description
audits/gates	table	–	–	Columns: gate(S|G|TQ|D), stage, pass(0/1), metrics_path.
audits/dov_tiling	float64[...]	–	[grid]	DoV tiling and boundary margins.
audits/manifest_hash	string	–	–	Cryptographic hash of manifest subset.
provenance/edges	table	–	–	(from_path, to_path, reason, tool, version, time_utc).
provenance/hashes	table	–	–	(path, sha256) for major nodes.

Key	Type	Units	Axes	Description
name	string	–	–	Scheme id (MSbar, MOM, ...).
loop_order	uint8	–	–	Loop order used (1,2,3,...).
mu_ref	float64	energy	–	Reference scale for matching/anchors.
decoupling	string	–	–	Decoupling prescription (step/smooth/...).
matching	string	–	–	EFT matching style (tree/1-loop/...).
notes	string	–	–	Free-form notes (optional).

/windows

/maps

/neutrino

/astro

/coords

/audits and /provenance

/rg/scheme

/rg/thresholds

Column	Type	Units	Axes	Description
species	string	–	–	Particle/field label.
m_decouple	float64	energy	–	Decoupling scale.
valid_low	float64	energy	–	Lower validity bound.
valid_high	float64	energy	–	Upper validity bound.
method	string	–	–	Matching formula id.
active	uint8	–	–	1=used, 0=ignored.

/rg/linkage

Key	Type	Units	Axes	Description
g123_flows	float64[3]	–	[i]	$\widehat{g}_i(\mu_{\rm ref})$ from flows.
g123_ref	float64[3]	–	[i]	$g_i^{\rm ref}$ from /sm_map.
residuals	float64[3]	–	[i]	$r_i = \widehat{g}_i - g_i^{\rm ref}$.
eps_abs	float64	–	–	Absolute tolerance.
eps_rel	float64	–	–	Relative tolerance.
score	float64	–	–	$\rho_{\rm RG}$ as defined above.
pass	uint8	–	–	1=pass, 0=fail.

Key	Type	Units	Axes	Description
bundle/replay	string	–	–	One-command replay script + checksums.
bundle/env_yaml	string	–	–	Environment lock (human-readable snapshot).
runs/inputs	string	–	–	YAML/JSON configs (machine- and human-readable).
runs/logs	string	–	–	Timestamped execution logs.

Key	Type	Units	Axes	Description
names	string[n_C]	–	[a]	Constraint labels $\{C_a\}$.
rho_br	float64	–	–	Max normalized bracket residual.
rho_Jac	float64	–	–	Max normalized Jacobi residual.
eps_abs	float64	–	–	Absolute tolerance.
eps_rel	float64	–	–	Relative tolerance.
grid_spec	string	–	–	Discretization spec (dt, dx, scheme).
batch_size	uint32	–	–	Number of tested configurations M.
R_br	float64[n_C,n_C]	–	[a,b]	$\max_m R_{ab}^{(m)}$ (optional detail).
J_Jac	float64[n_C,n_C,n_C]	–	[a,b,c]	$\max_m J_{abc}^{(m)}$ (optional).
pass	uint8	–	–	1=pass, 0=fail.

Key	Type	Units	Axes	Description
names	string[n_C]	–	[a]	Constraint labels.
types	string[n_C]	–	[a]	{first_class, second_class}.
structure	string	–	–	Symbolic ${F_{ab}}^c$ or reference to notebook.

Key	Type	Units	Axes	Description
lagrangian_id	string	–	–	Root hash computed from /action/terms.
missing_action_hash	table	–	–	Paths lacking action_hash or mismatching it.
missing_depends_on	table	–	–	Paths lacking dependency on /action/terms.
pass	uint8	–	–	1=all downstream nodes consistent; 0 otherwise.

`/bundle` and `/runs`

`/audits/constraint_closure`

`/action/constraints` (optional)

`/audits/sot`

RG scheme and thresholds (operational spec)

We record the renormalization scheme, loop order, reference scale, and decoupling prescription under `/rg/scheme`, and the EFT thresholds under `/rg/thresholds`. A linkage test compares the couplings $g_i(\mu_{\mathrm{ref}})$ obtained from `/rg/flows` to the YM anchor values stored in `/sm_map/g123_ref`.

RG linkage test.

Let $\widehat{g}_i(\mu_{\mathrm{ref}})$ be the interpolation of the flows at the reference scale and g_i^{ref} those in `/sm_map/g123_ref`. Define residuals $r_i = \widehat{g}_i - g_i^{\mathrm{ref}}$ and the score $\rho_{\mathrm{RG}} = \max_i \frac{|r_i|}{\max(\epsilon_{\mathrm{abs}}, \epsilon_{\mathrm{rel}}\,|g_i^{\mathrm{ref}}|)}$. Pass if $\rho_{\mathrm{RG}} \leq 1$.

Auditability.

Write results under `/rg/linkage` (see schema) and append a row to `/audits/gates` with `gate="S"`, `stage="rg"`, `metrics_path="/rg/linkage"`.

Chapter 25

Gate Automation and DoV Scans

25.1 Batch Gate runner: S/G/TQ/D

Purpose.

The *Batch Gate Runner* (BGR) executes, in a single sweep, all four consistency gates S/G/TQ/D across the declared DoV. It ingests a compiled `.h5` bundle (schema v7), evaluates analytic and numerical certificates, and emits machine–checkable pass/fail flags with diagnostic margins and failure artifacts.

Inputs/Outputs.

Item	Path / Symbol	Description
Input bundle	`msp_v7.h5`	Conforms to §24.
DoV tiling	`/audits/dov_tiling`	Grid of backgrounds/scales to scan.
Cone LMIs	`/cones/lmi/{mu_min,L_loc,v_front}`	Precomputed cone data.
GKSL block	`/quantum/gksl/{choi_eigs,choi_traces,di_bounds}`	Channel diagnostics.
Windows	`/windows/kill`	Decision/kill windows per observable.
Gate flags (out)	`/audits/{gate_S,gate_G,gate_TQ,gate_D}`	1/0 pass bits.
Logs/metrics (out)	`/audits/*`	Margins, violators, counters, hashes.

S–Gate (Spectral Positivity / Hyperbolicity)

For each DoV point, assemble principal symbols of the linearized system. Let $\mathsf{K}(\mathbf{x}), \mathsf{Z}(\mathbf{x}, \hat{n})$ be kinetic/gradient blocks. Define the symmetrized principal symbol

$$\mathsf{P}(\hat{n}) \;=\; \mathsf{K}^{-1/2}\,\mathsf{Z}(\hat{n})\,\mathsf{K}^{-1/2}, \qquad \lambda_{\min}^{(S)}(\hat{n}) = \lambda_{\min}\Big(\mathsf{P}(\hat{n})\Big).$$

Pass criterion:

$$\mathsf{K} \succeq 0, \quad \inf_{\hat{n}} \lambda_{\min}^{(S)}(\hat{n}) \;\geq\; 0, \quad \text{Hadamard preconditions satisfied on } (M, g_{\text{eff}}).$$

Record the worst margin $\delta_S = \inf_{\hat{n}} \lambda_{\min}^{(S)}(\hat{n})$ and the list of offending tiles if $\delta_S < 0$.

Constraint algebra closure (Gate S, stage="action")

Let $\{C_a(x)\}_{a=1}^{N_C}$ denote the first-class constraints on a Cauchy slice Σ_t (e.g., Gauss constraints for YM and diffeo/momentum constraints). On the discrete grid, define the Poisson bracket via the discrete symplectic form and test the closure

$$\{C_a, C_b\}_{\text{PB}} \;\approx\; F_{ab}{}^{c}\, C_c \quad \text{(on shell)},$$

where $F_{ab}{}^{c}$ are the structure functions (possibly field-dependent).

Metrics.

On a test batch of configurations $\{\phi^{(m)}\}_{m=1}^{M}$,

$$R_{ab}^{(m)} := \Big\| \{C_a, C_b\}_{\text{PB}} - F_{ab}{}^{c}\, C_c \Big\|_2, \tag{25.1}$$

$$J_{abc}^{(m)} := \Big\| \{C_a, \{C_b, C_c\}\} + \text{cyc.} \Big\|_2 \quad \text{(Jacobi)}. \tag{25.2}$$

Normalize by $D_{ab}^{(m)} = \max(\epsilon_{\text{abs}},\, \epsilon_{\text{rel}}\, \|C_a\|_2 + \|C_b\|_2)$ and $D_{abc}^{(m)} = \max(\epsilon_{\text{abs}},\, \epsilon_{\text{rel}}\, (\|C_a\|_2 + \|C_b\|_2 + \|C_c\|_2))$. Define the scores

$$\rho_{\text{br}} := \max_{m,a,b} \frac{R_{ab}^{(m)}}{D_{ab}^{(m)}}, \qquad \rho_{\text{Jac}} := \max_{m,a,b,c} \frac{J_{abc}^{(m)}}{D_{abc}^{(m)}}.$$

Acceptance.

Pass the gate if $\rho_{\text{br}} \leq 1$ and $\rho_{\text{Jac}} \leq 1$. Record tolerances $(\epsilon_{\text{abs}}, \epsilon_{\text{rel}})$, grid spec, and batch size.

Auditability.

Write results to `/audits/constraint_closure` (schema below) and append a row to `/audits/gates` with `gate="S"`, `stage="action"`, `metrics_path="/audits/constraint_closure"`.

RG linkage check (Gate S, stage="rg")

```
# Example CLI (pseudo):
python tools/check_rg_linkage.py --h5 msp_v7.h5 \
  --flows /rg/flows --scheme /rg/scheme \
  --smref /sm_map/g123_ref --write /rg/linkage \
```

```
  --eps_abs 1e-6 --eps_rel 5e-3

# Gate record:
# append to /audits/gates with gate="S", stage="rg",
# pass=(/rg/linkage/pass), metrics_path="/rg/linkage"
```

Constraint-closure check (Gate S, stage="action")

```
# Example CLI (pseudo):
python tools/check_constraint_closure.py --h5 msp_v7.h5 \
  --write /audits/constraint_closure \
  --eps_abs 1e-9 --eps_rel 1e-3 --batch 32

# Gate record:
# append to /audits/gates with gate="S", stage="action",
# pass=(/audits/constraint_closure/pass),
# metrics_path="/audits/constraint_closure"
```

SM-stage anomaly check (Gate S)

This target reads the matter-rep table under /sm_map/`charges`, computes $(A_{33Y}, A_{22Y}, A_{YYY}, A_{ggY})$ using the declared n_Y, writes /sm_map/`anomalies`, and appends a row to `/audits/gates` with `stage="sm"`.

```
# Example CLI (pseudo):
python tools/check_anomalies.py --h5 msp_v7.h5 \
  --charges /sm_map/charges/table \
  --write /sm_map/anomalies --epsA 1e-12 --nY from:/sm_map/nY_convention

# Gate record (stage="sm"):
# writes row to /audits/gates with gate="S", stage="sm",
# pass=(anomalies/pass), metrics_path="/sm_map/anomalies"
```

G–Gate (Causal Geometry / Cone LMIs)

Use precomputed cone LMIs (`/cones/lmi`) to check causal dominance and locality. With the gravity–cone LMI block $\mathsf{C}(\hat{n})$,

$$\mu_{\min}(\hat{n}) = \lambda_{\min}\Big(\mathsf{C}(\hat{n})\Big), \qquad r_{\text{front}} = \max_{\hat{n}} \frac{v_{\text{front}}(\hat{n})}{c_{\text{eff}}}.$$

Pass criterion: $\inf_{\hat{n}} \mu_{\min}(\hat{n}) \geq 0$ *and* $r_{\text{front}} \leq 1 + \varepsilon_G$. The locality radius L_{loc} is reported; if any instrument span exceeds L_{loc} the gate fails that tile.

TQ–Gate (Quantum Positivity / CPTP)

Let $\{\lambda_j^{\text{Choi}}\}$ be eigenvalues of the Choi matrix of each instrument/channel in `/quantum/gksl`.

$$\lambda_{\min}^{\text{Choi}} \geq -\varepsilon_{TQ}, \qquad \left|\text{Tr}(\Phi(\rho)) - \text{Tr}(\rho)\right| \leq \varepsilon_{\text{tr}},$$

with contractivity checks $\|\Phi(\rho) - \Phi(\sigma)\|_1 \leq \|\rho - \sigma\|_1$. *Pass criterion*: all channels in scope satisfy the above with declared margins $\varepsilon_{TQ}, \varepsilon_{\text{tr}}$.

D–Gate (Decision Windows / Kill Signatures)

For each observable entry in `/windows/kill` with bounds [`lower`, `upper`] at a stated CL, evaluate predictions in `/predictions`. *Pass criterion*: predicted values (or confidence envelopes) lie inside their windows; one–sided tests (e.g. EDM floors) obey the corresponding inequality. The gate returns the vector of per–observable decisions and highlights any *kill* events.

D-Gate (decision windows): two-sided rule

Given a prediction p for observable with `obs_id`, and its kill window $[L, U]$ recorded under `/windows/kill`, pass if $L \leq p \leq U$. Record p, (L, U), the p-value or distance-to-window, and write a row to `/audits/gates` with `gate="D"`, the appropriate `stage`, and `metrics_path`.

Execution order and parallelization

BGR executes gates in the order S → G → TQ → D, short–circuiting downstream gates on tiles already failed by upstream physics (e.g. no TQ on causality–violating tiles). Tiles are distributed across workers; random seeds for any stochastic diagnostic are fixed and stored under `/runs`.

Algorithm sketch (pseudo–code)

```
load H = h5open("msp_v7.h5")
for tile in H["/audits/dov_tiling"]:
  S = check_S_gate(tile, H["/action/terms"], H["/backgrounds"])
  if !S.pass: mark_fail(tile,"S"); continue
  G = check_G_gate(tile, H["/cones/lmi"])
  if !G.pass: mark_fail(tile,"G"); continue
  TQ = check_TQ_gate(tile, H["/quantum/gksl"])
  if !TQ.pass: mark_fail(tile,"TQ"); continue
  D = check_D_gate(tile, H["/predictions"], H["/windows/kill"])
```

```
  record(tile, S,G,TQ,D)
write_audits(H, aggregates, worst_margins, offenders)
```

Metrics written to `/audits`

- `gate_S`, `gate_G`, `gate_TQ`, `gate_D` (uint8).
- `margins/{delta_S, mu_min, r_front, choi_min, edm_delta, ...}`.
- `offenders/{tiles_S, tiles_G, channels_TQ, windows_D}` (indices/strings).
- `log`: human–readable summary (YAML/UTF-8).

Acceptance 25.1. Batch Gate runner The batch run *passes* iff (i) each scanned DoV tile passes S and G; (ii) all channels on surviving tiles satisfy TQ; (iii) predicted observables on surviving tiles satisfy D with their declared confidence levels; and (iv) all margins, seeds, hashes, and failure artifacts are written under `/audits` and reproducible from a one–command replay.

25.2 DoV tiling and boundary detection

Purpose.

We construct the *Domain of Validity* (DoV) as a subset of the parameter space $\Theta \subset \mathbb{R}^d$ by exhaustively scanning tiles, evaluating the four gates S/G/TQ/D, and locating the *boundary* where any gate transitions from pass to fail. The output is a machine–auditable partition together with a high–fidelity approximation of the boundary hypersurface.

Parameter space, tiling, and scaling

Let $\theta = (\theta_1, \ldots, \theta_d) \in \Theta = \prod_{j=1}^{d} [a_j, b_j]$. We choose affine or logarithmic reparametrizations $\vartheta_j = \psi_j(\theta_j)$ to equalize dynamic ranges and tile $\Psi(\Theta)$ by hyperrectangles $\mathcal{C}_\alpha$ of edge lengths h_j. Each tile carries:

$$\texttt{status}(\mathcal{C}_\alpha) \in \{\texttt{pass}, \texttt{fail}, \texttt{mixed}\}, \qquad \texttt{margins}(\mathcal{C}_\alpha) = \{\delta_S, \mu_{\min}, \delta_{TQ}, \delta_D\}_\alpha.$$

Gate score fields and unified margin

For a point θ, define scalar *gate scores*:

$$\delta_S(\theta) = \inf_{\hat{n}} \lambda_{\min}^{(S)}(\hat{n}; \theta), \quad \delta_G(\theta) = \inf_{\hat{n}} \mu_{\min}(\hat{n}; \theta), \quad \delta_{TQ}(\theta) = \lambda_{\min}^{\text{Choi}}(\theta) + \varepsilon_{TQ},$$

and the front–speed slack

$$\delta_{\text{front}}(\theta) = \Big(1 + \varepsilon_G\Big) - r_{\text{front}}(\theta), \qquad r_{\text{front}} = \max_{\hat{n}} \frac{v_{\text{front}}(\hat{n};\theta)}{c_{\text{eff}}(\hat{n};\theta)}.$$

For the decision windows (D) with observable vector $O(\theta)$ and admissible set W at stated CL, define the *signed distance*

$$\delta_D(\theta) = \begin{cases} +\text{dist}\Big(O(\theta),\, \partial W\Big), & O(\theta) \in W, \\ -\text{dist}\Big(O(\theta),\, \partial W\Big), & O(\theta) \notin W. \end{cases}$$

Normalize scores by declared tolerances $\tilde{\delta}_S = \delta_S/\varepsilon_S$, $\tilde{\delta}_G = \min(\delta_G, \delta_{\text{front}})/\varepsilon_G$, $\tilde{\delta}_{TQ} = \delta_{TQ}/\varepsilon_{TQ}$, $\tilde{\delta}_D = \delta_D/\varepsilon_D$, and define the *unified margin*

$$m(\theta) = \min\Big\{\tilde{\delta}_S, \tilde{\delta}_G, \tilde{\delta}_{TQ}, \tilde{\delta}_D\Big\}.$$

Then the indicator $g(\theta) = \mathbb{1}_{\{m(\theta)\geq 0\}}$ marks pass/fail and the boundary is the zero–level set $\partial\text{DoV} \approx \{\, \theta \mid m(\theta) = 0 \,\}$.

Sampling policy and adaptive refinement

Start with a space–filling design (Latin Hypercube or sparse grid) of N_0 points. Evaluate m. Fit a surrogate $\widehat{m}$ (Gaussian Process or RBF) that provides mean/variance (μ, σ). Iterate batches of size B by maximizing an acquisition function concentrated near the boundary and in high–uncertainty regions:

$$\mathcal{A}(\theta) = \exp\left(-\frac{\mu(\theta)^2}{2\sigma(\theta)^2}\right) \cdot \Big(\sigma(\theta) + \sigma_0\Big).$$

Flag any tile $\mathcal{C}_\alpha$ with $\min_{\mathcal{C}_\alpha} m < 0$ and $\max_{\mathcal{C}_\alpha} m > 0$ as `mixed` and *bisect* it (adaptive refinement) until a geometric tolerance is met.

Boundary tracing and certification

Within each `mixed` tile, locate sign–changes of m on edges and apply marching–squares/cubes (in 2D/3D) or facet–projection (for $d > 3$) to construct boundary simplices. For linear edge interpolation between samples θ_a, θ_b with $m_a m_b < 0$,

$$\theta^\star = \theta_a + \frac{|m_a|}{|m_a| + |m_b|}\,(\theta_b - \theta_a).$$

Estimate $\|\nabla m\|$ by local finite differences; if a Lipschitz bound L is estimated, any tile with diameter $\text{diam}(\mathcal{C}_\alpha) < |m_c|/L$ at its center c is certified `pass`/`fail` without further

splits.

Stopping rules and quality targets

Stop when *all* hold:

(i) Max tile diameter in the boundary band $\{\, |m| \le \tau_m \,\}$ is $\le h_{\max}$.

(ii) Surrogate cross–validation RMSE on m is $\le \epsilon_{\mathrm{CV}}$.

(iii) Hausdorff distance between successive boundary meshes $\le \epsilon_H$.

HDF5 outputs for DoV scans (v7)

The runner must write the following groups under `/audits`. All numeric datasets carry attributes `units`, `axes`, `depends_on` (upstream paths), and `action_hash` (copy of root `lagrangian_id`).

- `/audits/dov_tiling`:
  - `centers` float64[n_tiles,d] (axes=[tile,dim])
  - `sizes` float64[n_tiles,d] (tile box lengths)
  - `status` uint8[n_tiles] (0=unvisited, 1=pass, 2=fail, 3=mixed)
  - `raw_delta` float64[n_tiles] (raw δ-margins)

  Attributes: `scaling={affine|log}`, tolerances ε_*.
- `/audits/dov_boundary`: `nodes` float64[n_v,d], `simplices` int32[n_s,d+1], `labels` uint8[n_v] (active trigger code: 1=gap-closure, 2=CP-loss, 3=cone-violation, 4=window-break). Attribute `legend` documents the mapping.
- `/audits/dov_surrogate`: `type` (string), `hyperparams` (table), `cv_metrics` (table: RMSE, R^2, ...).
- `/audits/dov_stopping`: scalars ϵ_H, $h_{\max}$, ϵ_{CV}, N_{eval} with `units` and declared thresholds.

Pseudo–code

```
init design D0  ⊂ Θ
eval m on D0; fit surrogate m̂
repeat
  pick batch B = argmax_A 𝒜(θ) over candidate set
  eval m on B; update surrogate m̂
  refine tiles with sign-change of m (adaptive bisection)
```

```
  extract boundary mesh by marching hypercubes
until (h_max, ε_CV, ε_H) tolerances satisfied
write HDF5 artifacts under /audits/*
```

Checklist 25.1: DoV tiling acceptance

- **Coverage.** Every tile is labeled `pass`/`fail`/`mixed`; no unlabeled gaps; sampling log and seeds recorded.
- **Boundary quality.** Reported mesh satisfies Hausdorff tolerance ϵ_H and lists the active trigger per facet.
- **Reproducibility.** `/audits/dov_surrogate` and `/audits/dov_stopping` exist with hashes; replay reproduces labels.

AR.

CI workflows instantiate nightly DoV scans on a coarse grid and weekly adaptive scans with saved `*.h5` failure artifacts and dashboard plots (cone margins, Choi spectra, window distances) hyperlinked to the corresponding tiles.

Chapter 26

Public Bundle, APIs, and Reproducibility

26.1 One-command rebuild; replay logs

Goal.

Provide a *single entry point* that reconstructs all public artifacts (the HDF5 pack, cards, figures, tables) from versioned inputs and emits machine–checkable replay logs. The rebuild must be deterministic on the declared DoV: random seeds, solver tolerances, grids, and Gate thresholds are pinned and recorded.

Public bundle layout

A release bundle is a compressed archive `usft_v7_public.tar.gz` containing (illustrative):

- `msp_v7.h5` (canonical HDF5 file, cf. §**??**);
- `cards/` (theorem/prediction one–pagers, PDFs/TeX);
- `notebooks/` (reproducible figures/tables; no internet calls);
- `configs/` (YAML manifests for runs, RG, DI, ND–FD, cones);
- `replay.sh` & `Makefile` (one–command wrapper);
- `env.lock` (exact toolchain lock: compiler, Python, packages);
- `SHA256SUMS` (checksums of all top–level artifacts).

One–command interface

Users rebuild everything via either the shell script or `make` target:

```
# one-command replay (script)
./replay.sh --bundle ./usft_v7_public.tar.gz --verify --plots

# equivalent make target
make replay BUNDLE=./usft_v7_public.tar.gz VERIFY=1 PLOTS=1
```

Replay protocol (scripted steps).

The driver executes the following stages in order; each stage appends a timestamped record to the replay log and to the HDF5 `/audits` branch.

R1. Unpack & verify. Expand the bundle, compute SHA256, and compare against `SHA256SUMS`. Abort on any mismatch.

R2. Recreate environment. Instantiate the locked toolchain (`env.lock`); pin RNG seeds and number of threads.

R3. Rebuild numerics. Run background solvers, linear modes, soliton packs, and DI/GKSL calibrations using the provided YAML configs (no network access). Write intermediate HDF5 shards under `./build/`.

R4. Assemble pack. Merge shards into `msp_v7.h5` with units, axes, and provenance; populate `/audits/`.

R5. Gate audit. Evaluate S/G/TQ/Don the rebuilt HDF5; compare to reference margins stored in the bundle; flag deltas.

R6. Figures/cards. Regenerate plots and one–page cards from the rebuilt HDF5; compare checksums against shipped PDFs (tolerating metadata noise).

R7. Finalize replay log. Emit human–readable `replay.log` and embed a structured copy under HDF5 (paths below).

Minimal `replay.sh` (illustrative)

```
#!/usr/bin/env bash
set -euo pipefail
BUNDLE="${1:-usft_v7_public.tar.gz}"

echo "[1/7] Unpack & verify"
tar -xzf "$BUNDLE"
sha256sum -c SHA256SUMS

echo "[2/7] Environment"
# example: conda/mamba + lockfile
```

```
conda env create -f env.lock -n usft-v7 || true
conda activate usft-v7
export OMP_NUM_THREADS=1 MKL_NUM_THREADS=1 PYTHONHASHSEED=0

echo "[3/7] Rebuild numerics (no internet)"
python tools/build_backgrounds.py   --cfg configs/backgrounds.yaml
python tools/build_linear_modes.py  --cfg configs/linear.yaml
python tools/build_solitons.py      --cfg configs/solitons.yaml
python tools/build_gksl.py          --cfg configs/gksl.yaml

echo "[4/7] Assemble HDF5"
python tools/assemble_h5.py --out msp_v7.h5 --from ./build

echo "[5/7] Gate audit"
python tools/audit_gates.py --h5 msp_v7.h5 --ref configs/gates_ref.yaml

echo "[6/7] Figures/cards"
python tools/make_figures.py --h5 msp_v7.h5 --out ./figures
python tools/make_cards.py   --h5 msp_v7.h5 --out ./cards

echo "[7/7] Replay log"
python tools/embed_replay.py --h5 msp_v7.h5 --log replay.log
echo "DONE"
```

Reference `Makefile` targets (excerpt)

```
.PHONY: replay gates figures cards clean

replay: env gates figures cards
 @echo "Replay completed."

env:
 conda env create -f env.lock -n usft-v7 || true

gates:
 python tools/assemble_h5.py --out msp_v7.h5 --from ./build
 python tools/audit_gates.py --h5 msp_v7.h5 --ref configs/gates_ref.yaml
gates:
    python tools/assemble_h5.py --h5 msp_v7.h5 --from ./build
```

```
    python tools/check_anomalies.py --h5 msp_v7.h5 \
        --charges /sm_map/charges/table \
        --write /sm_map/anomalies --epsA 1e-12 --nY from:/sm_map/nY_convention
    python tools/audit_gates.py --h5 msp_v7.h5 --append
    gates:
\tpython tools/check_anomalies.py --h5 msp_v7.h5 --charges /sm_map/charges/table \
\t  --write /sm_map/anomalies --epsA 1e-12 --nY from:/sm_map/nY_convention
\tpython tools/check_rg_linkage.py --h5 msp_v7.h5 --flows /rg/flows \
\t  --scheme /rg/scheme --smref /sm_map/g123_ref --write /rg/linkage \
\t  --eps_abs 1e-6 --eps_rel 5e-3
\tpython tools/audit_gates.py --h5 msp_v7.h5 --append
gates:
\tpython tools/check_constraint_closure.py --h5 msp_v7.h5 \
\t  --write /audits/constraint_closure --eps_abs 1e-9 --eps_rel 1e-3 --batch 32
figures:
 python tools/make_figures.py --h5 msp_v7.h5 --out ./figures

cards:
 python tools/make_cards.py --h5 msp_v7.h5 --out ./cards
```

Reference YAML configs (excerpt)

```
# runs/inputs/rg_scheme.yaml
name: MSbar
loop_order: 2
mu_ref: 91.1876        # GeV
decoupling: step
matching: tree
notes: "SM-like running with step decoupling at thresholds."
```

```
# runs/inputs/rg_thresholds.yaml
- species: top
  m_decouple: 172.5    # GeV
  valid_low: 100.0
  valid_high: 1.0e5
  method: step
  active: 1
- species: Higgs
  m_decouple: 125.25
  valid_low: 80.0
```

```
  valid_high: 1.0e5
  method: step
  active: 1
```

Reference mini-bundle: `msp_v7_ref.h5`

Goal: a minimal public HDF5 that reproduces one precise prediction and passes a published kill window (Gate D).

Contents (minimal).

- Root attrs: `schema_version="v7"`, `build_time_utc`, `git_commit`, `env_lock`, `units_system`, `lagrangian_id`, `graph_hash`.
- `/sm_map`: `K,O,alpha_tilde,nY_convention,g123_ref,thetaW_ref, charges,anomalies (pass)`.
- `/rg`: `flows,scheme,thresholds` + `/rg/linkage` (pass).
- `/predictions/ew/thetaW_MZ`: scalar $\sin^2\theta_W(M_Z)$ with `obs_id="thetaW_MZ"`.
- `/windows/kill`: entry for `thetaW_MZ` with $[L, U]$ (95% CL).
- `/audits/gates`: rows for S (sm, rg) and D (predictions).
- `/bundle/replay`: script + checksums; `/runs/inputs`: YAML snapshots.

Replay protocol (pseudo).

```
# replay_thetaW_MZ.py (pseudo)
h5 = open("msp_v7_ref.h5","r+")
mu_ref = h5["/rg/scheme/mu_ref"]
gflows = interpolate(h5["/rg/flows"], mu=M_Z)        # -> \hat g_i(M_Z)
g1,g2 = gflows[1], gflows[2]
thetaW = g1**2/(g1**2 + g2**2)
write(h5, "/predictions/ew/thetaW_MZ", thetaW,
      attrs={"obs_id":"thetaW_MZ","units":"-","axes":"[]",
             "depends_on":["/rg/flows","/rg/scheme"]})
# D-Gate:
L = h5["/windows/kill/lower"]; U = h5["/windows/kill/upper"]
passed = int(L <= thetaW <= U)
append_gate(h5, gate="D", stage="predictions", pass=passed,
            metrics_path="/predictions/ew/thetaW_MZ")
```

SoT verification (hash + propagation)

```
# Example CLI (pseudo):
python tools/compute_lagrangian_id.py --h5 msp_v7.h5 \
  --root /action/terms --write-root-attr lagrangian_id

python tools/verify_sot.py --h5 msp_v7.h5 \
  --expect-root lagrangian_id --check-propagation \
  --check-depends-on /action/terms --write /audits/sot

# Writes /audits/sot with fields:
#   lagrangian_id (string), missing_action_hash (table of paths),
#   missing_depends_on (table), pass (uint8)
# Adds a row to /audits/gates with gate="S", stage="action",
# metrics_path="/audits/sot".
```

Makefile target (excerpt).

```
gates:
  python tools/check_anomalies.py --h5 msp_v7_ref.h5 --write /sm_map/anomalies ...
  python tools/check_rg_linkage.py --h5 msp_v7_ref.h5 --write /rg/linkage ...
  python tools/replay_thetaW.py   --h5 msp_v7_ref.h5
```

Reference YAML configs (thetaW window)

```
# runs/inputs/thetaW_window.yaml
obs_id: thetaW_MZ
bound_type: two-sided
lower: 0.2305
upper: 0.2320
confidence: 0.95
```

HDF5 replay keys (v7)

On successful replay, the script augments the HDF5 file with:

- `/bundle/manifest` (string): list of artifact filenames with SHA256.
- `/bundle/replay/log` (string): the exact `replay.log`.
- `/bundle/replay/metrics` (datasets): gate margins, figure hashes, timing, CPU/GPU info, solver versions.
- `/runs` (group): copies of the YAML configs and solver/grid manifests.

- `/audits/gate_S`, `/audits/gate_G`, `/audits/gate_TQ`, `/audits/gate_D` (uint8 flags + details).

Checklist 26.1: Rebuild acceptance

- **Checksum integrity.** All shipped artifacts match `SHA256SUMS`.
- **Determinism.** Rebuilt `msp_v7.h5` reproduces reference gate margins within declared tolerances; figure/card hashes match (ignoring PDF metadata).
- **Audit completeness.** Paths under `/bundle/replay/` and `/audits/` exist and contain the required fields; `env.lock` fingerprint is embedded at the HDF5 root.

Notes on portability.

The one–command replay is OS–agnostic provided the locked toolchain is honored. Container recipes (Docker/Singularity) may be distributed alongside the bundle; however, network access is disabled by default during `replay.sh` to guarantee closed–world reproducibility.

26.2 API for analysis and replay

Purpose.

Expose a minimal, stable interface that: (i) loads the canonical HDF5 pack, (ii) queries predictions, gates, and metadata, (iii) regenerates figures/cards, and (iv) replays the full build deterministically. The API is read–only with respect to shipped artifacts; any derived results are written to a user-specified output directory and, when appropriate, appended under `/bundle/replay/*` inside a working copy of the HDF5 file.

Public Python API (minimal surface)

The reference implementation ships as `usft/api.py`. The core calls are:

- `open_pack(path) -> h5`: open HDF5 (read-only) and verify `schema_version=="v7"`.
- `info(h5) -> dict`: summarize keys, units system, gates, DoV tiles.
- `get_gauge_couplings(h5, mu="ref") -> dict`: read (g_1, g_2, g_3) at μ_{ref} or a requested scale.
- `get_weinberg(h5, scale="MZ") -> float`: return $\sin^2\theta_W$ at M_Z (RG-transported if needed).
- `list_kill_windows(h5) -> list[dict]`: enumerate per-observable decision windows.

- `check_gates(h5) -> dict`: report pass/fail and margins for S/G/TQ/D.
- `make_figures(h5, outdir)`: regenerate reference plots from the pack (no internet).
- `replay(bundle_tar, verify=True, plots=False) -> dict`: one-command rebuild wrapper (cf. §26.1).

Example (analysis session).

```
from usft.api import open_pack, info, get_gauge_couplings, get_weinberg, \
                     list_kill_windows, check_gates

h5 = open_pack("msp_v7.h5")
meta = info(h5)
print("Units:", meta["units_system"], "Gates:", meta["gates"])

g = get_gauge_couplings(h5, mu="ref")     # {'g1':..., 'g2':..., 'g3':...}
s2w = get_weinberg(h5, scale="MZ")        # sin^2(theta_W) at M_Z
windows = list_kill_windows(h5)           # [{'obs_id':'cT', 'lower':..., ...}, ...]
gates = check_gates(h5)                   # {'S':{'pass':1,'margin':...}, 'G':..., 'T
```

CLI (scriptable interface)

A thin `usft` command mirrors the API for shell workflows:

```
# summarize pack and gates
usft info --h5 msp_v7.h5 --json

# regenerate figures and cards to ./out
usft figures --h5 msp_v7.h5 --out ./out

# run gate audit vs reference tolerances
usft gates --h5 msp_v7.h5 --ref configs/gates_ref.yaml

# one-command replay (cf. Section V7-rebuild)
usft replay --bundle usft_v7_public.tar.gz --verify --plots
```

HDF5 access conventions

All access uses absolute paths within the pack (see §??). Key lookups include:

- Predictions: `/predictions/sin2thetaW_MZ`, `/predictions/STU`, `/predictions/cT`.
- Couplings/RG: `/rg/flows/g123`, `/rg/flows/mu`.

- Gates/Audits: `/audits/gate_S`, `/audits/gate_G`, `/audits/gate_TQ`, `/audits/gate_D`.
- Windows: `/windows/kill/` records with `obs_id, bound_type, lower, upper, confidence`.
- Provenance: root attributes `schema_version`, `env_lock`, `git_commit`, `units_system`.

Error model and return codes

The API raises structured exceptions with machine-readable payloads:

- `SchemaError`: missing or incompatible `schema_version`.
- `KeyError`: absent group/dataset path (reports nearest prefix).
- `UnitsError`: missing or inconsistent `units`/`axes` attributes.
- `GateError`: audit fails to evaluate (malformed tolerances or missing metrics).
- `ReplayError`: bundle integrity or environment lock mismatch.

CLI returns nonzero exit codes mapped 1:Schema, 2:Key, 3:Units, 4:Gate, 5:Replay.

Versioning and compatibility

The `schema_version` at the HDF5 root must be `"v7"`. Minor additions that do not break the public keys (§**??**) are backward compatible. Any change to dataset shapes or semantics bumps the schema and the API advertises an explicit compatibility matrix in `usft.api.compat`.

Acceptance.

Independent replay reproduces baseline gates and predictions.

Part X

Global Milestones, Acceptance, and Artifacts

Chapter 27

Milestones, Exit Criteria, and Artifacts

Preface. This part converts the scientific stack into an operational program with auditable deliverables. Each milestone M_k couples (i) *what* must be produced (artifacts, proofs, figures) with (ii) *how* it is accepted (gate certificates, numerical checks, and DoV audit). Gates S/G/TQ/Dare enforced at the point where their assumptions first become meaningful and are re-checked downstream.

27.1 Global Milestones (M1–M12)

M1. MSP*/DoV freeze (Charter signed; Theorem 1.2).
Deliverables: Signed Charter; MSP* manifest (fixed $\kappa_G, \mathcal{N}_{u,d,\ell}$, topology); DoV sheet with triggers.
Exit: A1–A3 satisfied; baseline Gate certificates attached (hashes + timestamps).

M2. Spectral action compiled (EL/Noether; Theorem 2.1).
Deliverables: $\mathcal{L}$ blocks with units; EL equations; Noether currents; stress tensor $T_{\mu\nu}$.
Exit (S/G): cone LMIs $\mu_{\min} \geq 0$; forward-limit positivity matrices $\succeq 0$.

M3. PDE well-posedness (CFL/consistency; convergence).
Deliverables: Symmetrizable hyperbolic form; CFL maps; order-of-accuracy (GCI) tables.
Exit: CFL bound respected; observed order within $\pm 20\%$ of method order; reproducible runs.

M4. Soliton pack (energies/topology/S/G).
Deliverables: Minimization logs; topology certificates; Hessian spectra.
Exit: Energy/charge convergence; ghost-free Hessians; cone constraints satisfied.

M5. Linear modes/cones (LMI margins; $v_{\text{front}} \leq \text{cEff}$).

Deliverables: $(\mathsf{K}, \mathsf{Z}, \mathsf{M}^2)$ tables; dispersion/cones; LMI grids.
Exit (G): $\mu_{\min} \geq 0$ on DoV tiles; measured front speeds $\leq$ cEff.

M6. CCR/CAR & Hadamard (GNS seeds; WF-set checks).
Deliverables: BRST quotient proof notes; Hadamard diagnostics (wave-front sets); GNS seed states.
Exit (S): positivity/microcausality checks pass; Cauchy-surface independence verified.

M7. GKSL & no-signalling (CPTP margins; DI regions).
Deliverables: Calibrated GKSL channels; Choi spectra; DI feasibility regions.
Exit (TQ): CPTP margins nonnegative; contractivity; no-signalling theorem conditions met.

M8. GR/SM recoverability (EFT matches).
Deliverables: Low-energy maps $\mathcal{L} \to \mathcal{L}_{\mathrm{GR}} + \mathcal{L}_{\mathrm{SM}}$; FRW sector; matching tables.
Exit: Baseline observables reproduced within tolerances; dimensional consistency audits pass.

M9. Signature pack (CMB/LSS/GW/precision/tabletop + kill windows).
Deliverables: Prediction envelopes; per-observable kill/decision windows; likelihood hooks.
Exit (D): Windows published with confidence levels; orthogonal templates attached.

M10. Inference pack (likelihoods, forecasts, Bayes factors).
Deliverables: Likelihood suite; priors; Fisher/MCMC outputs; posterior summaries.
Exit: Reproducible posteriors; decision tables mapping deltas to revision paths.

M11. Experiment pack (protocols + astro likelihoods).
Deliverables: EW/EDM/NSI/GW/CMB/LSS protocols; ND–FD maps; device-calibrated templates.
Exit: External-collab–ready specs; data paths and QA checks included.

M12. Release & CI (public HDF5 v7; one-command replay).
Deliverables: `msp_v7.h5`; cards/UFO; notebooks/API; CI logs and lockfiles.
Exit: Independent replay reproduces gates and predictions; bundle signatures verified.

27.2 Acceptance Gates and Decision Windows

We tie each milestone to quantitative, machine-checkable metrics grouped under S (spectral stability), G (causal geometry), TQ (quantum positivity), and D (dissipation/decision). A run or derivation *passes* when all required gate margins are nonnegative within the declared tolerances and the associated decision windows are satisfied.

Gate metrics (definitions and pass criteria)

S — Spectral stability / hyperbolicity.

$$\text{(S1) Kinetic positivity: } \lambda_{\min}(\mathsf{K}) \;\geq\; -\varepsilon_S, \tag{27.1}$$

$$\text{(S2) Gradient positivity: } \lambda_{\min}(\mathsf{Z}) \;\geq\; -\varepsilon_S, \tag{27.2}$$

$$\text{(S3) Forward-limit positivity (EFT blocks): } \mathcal{C}^{(2k)} \succeq -\varepsilon_S\,\mathbb{I}, \quad k = 1, 2, \ldots \tag{27.3}$$

Pass if all three hold on the declared DoV tiles.

G — Causal geometry / locality.

$$\text{(G1) Cone LMI margin: } \mu_{\min} \;\equiv\; \lambda_{\min}\Big(\mathcal{M}_{\text{cone}}\Big) \;\geq\; 0, \tag{27.4}$$

$$\text{(G2) Front speeds: } \max_{\text{tiles}} \frac{v_{\text{front}}}{\text{cEff}} \;\leq\; 1 + \varepsilon_G, \tag{27.5}$$

$$\text{(G3) Locality radius: } L_{\text{loc}} \;\text{ finite and recorded in } \texttt{/cones/lmi/L_loc}. \tag{27.6}$$

Pass if (G1)–(G3) hold across the DoV.

TQ — Complete positivity, trace, contractivity.

Let $\mathcal{E}$ be a quantum channel fitted from simulations, with Choi matrix $\mathsf{J}(\mathcal{E})$.

$$\text{(TQ1) CPTP: } \lambda_{\min}\Big(\mathsf{J}(\mathcal{E})\Big) \;\geq\; -\varepsilon_{TQ}, \quad \Big\|\mathcal{E}^\dagger(\mathbb{I}) - \mathbb{I}\Big\| \leq \varepsilon_{\text{TP}}, \tag{27.7}$$

$$\text{(TQ2) Contractivity: } \Big\|\mathcal{E}(\rho) - \mathcal{E}(\sigma)\Big\|_1 \;\leq\; \Big\|\rho - \sigma\Big\|_1 + \varepsilon_{\text{ctr}}, \tag{27.8}$$

$$\text{(TQ3) DI feasibility (no-signalling): } \widehat{\boldsymbol{O}} \;\in\; \mathcal{R}_{\text{DI}} \;\text{ (device-independent region under colored-noise model} \tag{27.9}$$

Pass if (TQ1)–(TQ3) hold.

D — Dissipation and decision layer.

$$\text{(D1) Nonnegative entropy production (discrete or continuous): } \sigma_{\text{diss}} \;\geq\; -\varepsilon_D, \tag{27.10}$$

$$\text{(D2) Energy ledgers: } \Delta E = \int (W_{\text{in}} - W_{\text{out}})\,dt \;\pm\; \varepsilon_E \text{ (closure)}, \tag{27.11}$$

$$\text{(D3) Decision windows: all reported observables lie within their published windows } \mathcal{W}_p. \tag{27.12}$$

Pass if (D1)–(D3) hold.

Decision windows and statistical acceptance

Let $\widehat{\boldsymbol{O}} \in \mathbb{R}^d$ denote a vector of predicted observables at a specified scale (e.g., μ_{ref} or M_Z), and $\boldsymbol{O}$ the corresponding measurements with covariance Σ.

Two-sided (ellipsoidal) acceptance.

$$\chi^2(\widehat{\boldsymbol{O}}) \;:=\; (\boldsymbol{O} - \widehat{\boldsymbol{O}})^\top \Sigma^{-1} (\boldsymbol{O} - \widehat{\boldsymbol{O}}) \;\le\; \chi^2_{d,\ \text{CL}}, \tag{27.13}$$

which defines the window $\mathcal{W}_p = \{\widehat{\boldsymbol{O}} : \chi^2 \le \chi^2_{d,\text{CL}}\}$.

One-sided (floor or upper-bound) acceptance.

For a scalar observable X with bound $X \ge X_{\text{floor}}$ (e.g. EDM floors) or $X \le X_{\text{cap}}$, accept if

$$\text{sgn} \cdot \frac{X - X_{\text{thr}}}{\sigma_X} \;\ge\; z_{\text{CL}}, \quad \text{with sgn} = +1 \text{ (floor)},\ \text{sgn} = -1 \text{ (cap)}. \tag{27.14}$$

ND–FD map consistency.

Given near/far spectra (n^N, n^F) and a modelled transfer T,

$$\chi^2_{NF} \;:=\; \left(n^F - T\, n^N\right)^\top \Sigma_{NF}^{-1} \left(n^F - T\, n^N\right), \quad \text{accept if } p_{NF} \ge p_{\text{gate}}. \tag{27.15}$$

Compact milestone–gate matrix

Milestone	Gates	Core metrics	Pass / Kill rule
M2: Spectral action	S, G	$\lambda_{\min}(\mathsf{K}), \lambda_{\min}(\mathsf{Z})$; forward $\mathcal{C}^{(2k)} \succeq 0$; cone LMI $\mu_{\min}$	Pass if all margins ≥ 0; Kill if any margin < 0.
M3: PDE well-posedness	S	CFL map; observed order (GCI); reproducibility logs	Pass if CFL holds & $p_{\text{obs}} \approx p_{\text{method}}$; Kill if violation.
M5: Modes/cones	G	$v_{\text{front}}/\text{cEff} \le 1 + \varepsilon_G$; $\mu_{\min} \ge 0$	Pass if both satisfied; Kill otherwise.
M7: GKSL/no-sig.	TQ	$\lambda_{\min}(\mathsf{J}) \ge -\varepsilon_{TQ}$; TP error $\le \varepsilon_{\text{TP}}$; DI feasibility	Pass if all; Kill on CPTP breach or DI infeasibility.
M8–M9: Recovery/Signatures	D	Energy/entropy ledgers; $\widehat{\boldsymbol{O}} \in \mathcal{W}_p$ (two-/one-sided); ND–FD p_{NF}	Pass if inside windows; Kill if outside or $p_{NF} < p_{\text{gate}}$.

Audit and reproducibility linkage

For each gate evaluation, store in HDF5: (i) the raw metric arrays (e.g. `/cones/lmi/mu_min`, `/quantum/gksl/choi_eigs`); (ii) the decision thresholds (attributes `eps_S`, `eps_G`, `eps_TQ`, `eps_D`); and (iii) a boolean flag per gate under `/audits/gate_S`, `/audits/gate_G`, `/audits/gate_TQ`, `/audits/gate_D`. Decision windows are serialized under `/windows/kill` (observable id, bound type, lower/upper, confidence).

Acceptance 27.1. Global acceptance (section 27.2) A milestone M_k is *accepted* iff all its required gate metrics pass within tolerances and every linked observable lies inside its published decision window. Otherwise M_k is *killed* and routed to the revision playbook.

27.3 Artifacts (Must-ship)

This section specifies the public deliverables required for a *valid* Volume VII release. Each artifact is versioned, checksummed, and linked to the S/G/TQ/Daudits so that independent teams can reproduce *both* the gate certificates and the headline predictions.

Packaging & versioning

- **Release tag:** `v7.<major>.<minor>.<patch>` (semantic).
- **Bundle name:** `usft-v7-<tag>.tar.gz` with a top-level folder `usft-v7-<tag>/`.
- **Checksums/signing:** ship `SHA256SUMS` and `SHA256SUMS.sig` (detached signature).

`msp_v7.h5` (single-source-of-truth)

A unit-annotated HDF5 file collecting inputs, equations, audits, and predictions. It *must* include at least the following keys (cf. §**??**):

Path	Type	Content (units/axes attrs mandatory)
`/msp/inputs`	group	κ_G, $\mathcal{N}_{u,d,\ell}$, topology indices.
`/action/terms`	group	Kinetic/potential/mixings, Rayleigh functional.
`/backgrounds/*`	group	Minkowski/FRW/BH sheets; cone plots; thresholds.
`/cones/lmi`	group	$\mu_{\min}$ grids, L_{loc}, v_{front}.
`/quantum/gksl`	group	Choi spectra, TP errors, DI region flags.
`/rg/flows`	group	Scales μ, β-maps, (g_1, g_2, g_3) tracks.
`/predictions`	group	$\sin^2\theta_W$, (S, T, U), $c_T(f)$, $w_{\text{eff}}(a)$, $g_*(k)$.
`/windows/kill`	table	Observable id, bound type, lower/upper, confidence.
`/audits`	group	Gate flags `gate_S/G/TQ/D`, DoV tiling.
`/runs`	group	Solver/grid YAML, CFL, tolerances, seeds, elapsed.

Cards/UFO (v7)

Concise human-readable PDFs (or Markdown) summarizing theorems, assumptions, windows, and numerical margins. One card per item:

- **Theorem cards:** Born-limit; No-signalling; BRST/Ward from MSP.
- **Prediction cards:** $g_{1,2,3}$ at μ_{ref}; $\sin^2\theta_W(M_Z)$; (S, T, U); $c_T(f)$; NSI textures; EDM floors.
- **Window cards:** explicit kill/decision windows with covariance assumptions.

Each card references precise HDF5 paths (e.g. `/predictions/thetaW_MZ`).

Notebooks/API

- **Notebooks:** `notebooks/rg.ipynb`, `notebooks/cones.ipynb`, `notebooks/di.ipynb`, `notebooks/ndfd.ipynb`, each rebuilding figures from `msp_v7.h5`.
- **API:** `api/usft.py` (*read-only*) with helpers `load_h5`, `cones()`, `choi()`, `rg()`, `windows()`.

CI logs

Machine logs for reproducibility and auditing:

- `ci/` with `gate_S,G,TQ,D.json`, `DoV_scan.json`, replay logs, environment lockfiles (`conda-lock.yml` or `requirements.txt`).
- Dashboard screenshots or JSON sources for pass/fail badges.

Release tree (normative)

```
usft-v7-<tag>/
 |-- msp_v7.h5
 |-- cards/
 |    |-- theorem_born.pdf
 |    |-- theorem_nosig.pdf
 |    |__pred_thetaW_MZ.pdf
 |-- notebooks/
 |    |-- rg.ipynb
 |    |-- cones.ipynb
 |    |-- di.ipynb
 |    |__ ndfd.ipynb
 |-- api/usft.py
 |-- ci/
 |    |-- gate_S.json
 |    |-- gate_G.json
 |    |-- gate_TQ.json
 |    |-- gate_D.json
 |    |-- DoV_scan.json
 |    |__ env_lock.yml
 |-- SHA256SUMS
 |__ SHA256SUMS.sig
```

Acceptance (Artifacts)

Checklist 27.1: Artifact acceptance

- **Completeness.** All paths in the release tree exist and open; HDF5 passes schema checks (§??).
- **Traceability.** Every figure/table in cards is regenerated by a shipped notebook that reads only `msp_v7.h5`.
- **Integrity.** `SHA256SUMS` matches; `SHA256SUMS.sig` verifies.
- **Reproducibility.** `api/usft.py` and notebooks replay baseline gates and predictions within declared tolerances; CI logs show pass.

One-command replay.

Vendors must be able to run:

```
$ usftv7 replay --bundle usft-v7-<tag>.tar.gz --out ./replay/
```

and reproduce gate flags and prediction numbers stored under `/audits` and `/predictions` within the reported uncertainty budgets.

Epilogue (V7): Operational Einstein

Statement. On its DoV, the USFT stack unifies geometry and gauge via MSP with minimal inputs; it *predicts* (not postdicts), and it dies cleanly on predeclared windows. What remains is not rhetoric but *execution*: numerics, fingerprints, release, reproduction, and the next kill-resistant prediction.

Consolidated Closure Set

- **Theorem layer.** Born-limit under spectral flooding; Local GKSL $\Rightarrow$ *no superluminal signalling* on (M, g_{eff}); Ward/BRST identities as consequences of MSP on the physical quotient.
- **Predictive layer.** Geometric anchor for (g_1, g_2, g_3) at μ_{ref}; RG transport to M_Z for $\sin^2\theta_W$; oblique parameters (S, T, U); irreducible EDM floors; NSI textures and device-independent (DI) feasibility regions; $c_T(f)$ and $w_{\text{eff}}(a)$ under declared backgrounds.
- **Operational layer.** Four gates S/G/TQ/D enforced; DoV audited and tiled; HDF5 v7 schema as single source of truth; one-command replay and CI-backed reproducibility.

Risks, Triggers, and Revision Ladders

Boundary triggers (end-of-validity).

Gap closure in pointer sectors; loss of complete positivity (negative Choi eigenvalues within margin); cone/LMI violation (causal-speed overshoot); anomaly or charge-quantization failures in the SM mapping; RG pathologies at thresholds.

Revision ladder.

1. *Local patch.* Tighten windows or recalibrate channels (preserve MSP/DoV).
2. *Structural patch.* Adjust embeddings/overlaps or hypercharge convention; re-run anomaly and RG continuity checks.

3. *Principle-level fork.* Amend minimal axioms (S0–S4) with explicit deprecation notes and new acceptance proofs.

Public Artifacts & Reproducibility Pledge

The release ships `msp_v7.h5`, cards (theorems/predictions/windows), notebooks, APIs, and CI logs. Independent teams should reproduce gate flags and headline observables *from the HDF5 file alone*, within published uncertainty bands, using:

```
usftv7 replay --bundle usft-v7-<tag>.tar.gz --out ./replay/
```

All figures and numbers in this volume are generated by notebooks that read only `msp_v7.h5`. Checksums and signatures ensure artifact integrity.

Operational Outlook

- **Near term.** Extend DI regions under colored noise; tighten EDM floors; push ND–FD orthogonal templates for neutrino baselines; expand $c_T(f, z)$ coverage.
- **Mid term.** Full-spectrum background scans (FRW/BH); soliton packs with scattering benchmarks; systematic RG uncertainty budgets attached to (g_1, g_2, g_3) and $\sin^2\theta_W$ predictions.
- **Long term.** Cross-portfolio decision tables spanning cosmology, GW, precision, and tabletop, with rolling kill windows and automatic revision playbooks.

Minimal Glossary (Operational)

- **MSP** (Master Spectral Principle): minimal axiom set compressing Vols. I–VI, yielding emergent g_{eff} and gauge connections.
- **DoV** (Domain of Validity): scales/backgrounds where gates pass; crossing triggers a revision.
- **Gates** S/G/TQ/D: spectral stability; causal geometry; quantum CP/tomography; dissipation/decision.

Closing. The *Operational Einstein* program is now a contract: declare inputs, publish artifacts, pass the gates, state the windows, face the data. Either the unified spectral field stands *within* its DoV and predicts—or it fails cleanly and teaches us where to look next.